Is a Monetary Union a Never-Ending Story?*

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Abstract

This paper extends the existing literature on the long-run sustainability of a monetary union using an optimal stopping framework. We assume that inflation is endogenous and money growth is the control variable. Under a particular condition on some parameters, the union goes on forever. Moreover, the effective breakdown of the union is governed by two critical thresholds: (i) a lower level for domestic inflation and (ii) an upper level for union's inflation. The optimal time for leaving a monetary union is the first time either domestic inflation goes down the former threshold or union's inflation goes over the latter threshold.

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1 Introduction

The unexpectedly rapid pace of the progress towards monetary unification in Europe, after the crisis of the Exchange Rate Mechanism in 1992-93, has given new impetus, both on the theoretical and on the empirical side, to research on monetary integration.

Modern work in this field dates back to Friedman (1953) seminal paper, making the case for a flexible exchange rate regime in an economy characterized by relevant price and wage rigidities.

The earlier literature on optimal currency areas (Mundell, 1961; and McKinnon, 1963) focused on various macroeconomic features minimizing the loss of nominal exchange rate flexibility as a shock-absorber mechanism: a high degree of factor mobility and wage/price flexibility, a high degree of openness of a country and a suitable diversification of its industrial structure. Subsequent theoretical refinements pointed out the drawbacks of a purely criteria-based approach (Ishiyama, 1975), paving the way to a better analytical treatment of the effects of the various shocks affecting the economy (symmetric vs. asymmetric; temporary vs. permanent; nominal vs. real). On the whole, these developments allowed to cast more firmly this literature inside a cost-benefit framework (Fleming, 1971; Corden, 1972; and Tower and Willet, 1976).

More recent contributions in this area emphasize credibility issues associated with the introduction of a common currency, analyzing how reputational factors extend the range of benefits arising from monetary integration. This literature draws heavily on the game-theoretic approach pioneered in Barro and Gordon (1983), outlining strategic interactions between the Central Bank and the private sector and the consequent inflationary bias produced by time consistent monetary policies. In close connection with some results stemming from the EMS literature (Giavazzi and Pagano, 1988; and Giavazzi and Giovannini, 1989), one key theoretical insight from this work is the effectiveness of a monetary union as an anti-inflationary commitment device (De Grauwe, 1994). This insight motivates the main features assigned to the newly established European Central Bank as an institutional body largely independent from political pressures and whose preferences closely mimic those of the more inflation-averse participating country (Germany). The main topics addressed in this literature are the net benefits accruing to the union's leader country (Germany) and various issues concerning the dynamic process of monetary integration. As regards the former topic, it has been maintained that the credibility gains arising for the monetary union as a whole do not come for free, since they require a disproportionate (and possibly destabilizing) degree of control over the common monetary policy on the part of the leader country (Alesina and Grilli, 1993). Turning to the latter issues, the path-dependence of the monetary integration process has motivated some skepticism about the feasibility of a multispeed monetary union, while further union enlargements have been shown to be crucially affected by the preferences of potential new entrants relative to the median of the already existing union (Alesina and Grilli, 1993).

Despite the massive research effort outlined above, the literature on monetary integration has almost entirely focused on the prerequisites allowing various countries to give up their monetary sovereignty, rather than exploring which macroeconomic factors ensure the long run sustainability of a currency union. More specifically, although theoretical work has at times addressed some instability issues, they have typically been confined to the transition phase preceding the formal introduction of a common currency whereas, once a monetary union is complete, this outcome has usually been regarded as a strictly irreversible process. Some recent contributions offer interesting examples of the destabilizing influences which are likely to arise along the transition phase. A speculative attack can occur whenever the competitiveness of a high-inflation country is largely eroded by nominal exchange rate stability, while this situation may be exacerbated by the actual behavior of various economic agents (Giovannini, 1990). Alternatively, a confidence crisis may develop when a country defending a fixed parity holds a very large stock of public debt, a substantial proportion of which has a short-term maturity (Giavazzi and Pagano, 1990). In this regard, even a system of irrevocably fixed exchange rates is not completely immune from speculative pressures since, as long as different national currencies coexist, further exchange rate changes can never be definitively ruled out, thus precluding perfect interest rate equalization across various currencies (Gros and Thygesen, 1992). The creation of a common currency, by contrast, is traditionally considered as a "once and for all" event, whose additional benefits with respect to a system of irrevocably fixed exchange rates (elimination of transaction and information costs, elimination of price discrimination, dynamic efficiency gains) are expected to last forever in the future (Gros and Thygesen, 1992).

The irreversible character assigned to the above process in the literature is a serious drawback, particularly when its key theoretical insights are confronted with real world developments. As revealed by the empirical evidence, the actual experience of formal currency unions among sovereign national governments is indeed remarkably mixed. While some of them displayed an impressive long-run stability and are currently still in operation (CFA Franc Zone, East Caribbean Currency Area), other attempts proved ultimately unsustainable.

Cohen (1993) provides a comparative analysis of six historical episodes along the nineteenth and twentieth centuries, in order to pinpoint which factors turned out to be crucial for the sustainability of the surviving unions. The comparative historical analysis carried out in Cohen (1993) is appealing, but the approach taken by this author does not rely on an explicit economic model. A narrative framework of this kind is therefore inherently prone to emphasize the influence of institutional factors.

In a more recent contribution, Strobel (2001) develops a formal two-country model outlining which conditions may eventually lead to a monetary union disintegration. The choice of monetary disintegration is analyzed inside an optimal stopping framework (Dixit and Pindyck, 1994; Øksendal, 2000), deriving a trigger value in relative inflation preference parameters above which this option will immediately be exercised. The main result achieved in Strobel (2001) is that, for standard parameter values, the magnitude of the above threshold is relatively high, making the occurrence of monetary disintegration a rather unlikely event. Moreover, a higher uncertainty about inflation preference parameters increases this critical boundary, thus lowering the likelihood that a country will opt to return to monetary independence, whereas a higher discount rate (i.e. policy-makers being more short-sighted) has the opposite effect. One questionable feature of this paper is the assumption that inflation preference parameters change continuously over time, following geometric Brownian motions. A latter weakness of this model is that it focuses exclusively on the government loss functions across alternative monetary regimes, without specifying any underlying macroeconomic structure.

The purpose of the present paper is to contribute to the literature exploring the long-run sustainability of a currency union. While retaining some connections with earlier research, this paper innovates upon the current literature in many respects. In line with Strobel (2001), we rely on an optimal stopping approach in order to examine the value of the option of monetary disintegration. Differently from the above paper, however, our contribution allows for an active monetary policy. In fact, while the inflation rate is given in Strobel, in our framework it follows a stochastic process whose drift and volatility depend on the money growth rate. The dynamic intertemporal problem faced by a policy-maker is thus analyzed inside a more sophisticated macroeconomic model, where inflation represents the forcing state variable. Since money supply growth is the only control variable in this setting, we highlight the effects induced by a crucial policy instrument .

The structure of the paper may conveniently be summarized as follows. Section 2 outlines the basic structure of the economy and the loss functions characterizing alternative monetary regimes. Section 3 solves the intertemporal optimization problem faced by the policy-maker. We compute the value function associated to this problem and the behaviour of the optimal inflation rate. Section 4 shows under which conditions a given country would optimally choose to return to monetary independence. Section 5 summarizes the main results and provides some suggestions for further research.

2 The model

In line with Strobel (2001) we rely on an optimal stopping approach to examine the value of the option of monetary disintegration. Although sharing this analytical framework, our model significantly improves upon the above contribution since we endogenize the dynamics of the inflationary process taking explicitly into account the possibility of controlling the inflation rate through monetary policy.

In what follows, we outline the basic structure of the model with reference to a single country pursuing an independent monetary policy (we are therefore implicitly assuming that this country is operating under a floating exchange rate regime). As explained at the end of this section, however, this theoretical framework can easily be extended to characterize the inflationary process and policy-maker's incentives under a monetary union between n participating countries.

The dynamics of the inflation rate is assumed to be driven by the following stochastic differential equation:

$$\begin{aligned} d\pi(t) &= k(m(t)-\pi(t))\,dt+\sigma(m(t)+\phi)\,dW(t), \\ \pi(t_0) &= \pi_0, \end{aligned} \tag{1}$$

where π denotes the inflation rate, m is the money growth rate, dW is the differential of a one-dimensional Wiener process, σ is a constant volatility term, and k and ϕ are two parameters.

This specification assumes that money growth affects both the first and the second moment of the inflation rate, and is consistent with a large body of macroeconomic literature exploring the determinants of inflationary processes and their variability over time.

The former term on the right-hand side of Equation (1) reflects the widely held consensus view according to which, in the long run, inflation is basically a monetary phenomenon. Actually, in this set up, the steady-state inflation rate coincides with the money growth rate.[1] The k parameter can thus be interpreted as measuring the opposite of the degree of price stickiness in the economy, i.e. the speed at which inflation converges to its long-run value driven by the money growth rate. When k is very low (tending towards zero), price rigidity is high, so that monetary policy has a very small impact on the short-run inflation rate. High values for k, conversely, denote a substantial degree of price flexibility: in this case monetary policy has a larger impact on inflation, even in the short run.

Turning to the latter additive term in Equation (1), inflation volatility does not simply reflect the influence of purely stochastic disturbances, but is also positively related to the money growth rate. Since the money growth rate directly affects inflation, the rationale behind this specification is the existence of a causal link from the inflation rate to inflation volatility.

This relationship has a well-established tradition in macroeconomic theory since Okun (1971) and Friedman (1977) seminal contributions. Whereas the arguments put forward by these authors were largely ad hoc,[2] subsequent work provided more rigorous underpinnings to the existence of a positive influence of inflation on inflation uncertainty. Ball (1992) offers a well-grounded motivation

[1] This result holds if we abstract from the effects induced by real output growth on money demand (alternatively, m can be interpreted as the rate of monetary expansion in excess over the rate of real output growth). In order to simplify the analysis, we also abstract from shifts in velocity or control errors for the money supply. The above assumptions greatly simplify the algebra, whereas a more complex specification of inflation dynamics would leave our basic results qualitatively unchanged.

[2] The basic idea in Okun (1971) and Friedman (1977) is that a highly inflationary environment increases agents uncertainty about future monetary policy. In such an environment, monetary policy tends to be more erratic, usually producing stop-and-go policies, since the authorities face a worse trade-off between disinflating and supporting real economic activity. Although the above arguments are intuitively appealing, neither Okun (1971) nor Friedman (1977) formalize them through a rigorous theoretical model.

to this link extending the Barro and Gordon (1983) positive theory of monetary policy in two directions. First, following Canzoneri (1985), the Central Bank is assumed not to control inflation perfectly: exogenous shocks cause low-inflation equilibria to break down occasionally, with the economy alternating between periods of high and low inflation. Second, following Alesina (1987), this model assumes two policy-maker types, with different inflation preferences, who stochastically alternate in power. These assumptions lead to a natural link between inflation and uncertainty. When actual and expected inflation are low, there is a consensus that monetary policy will try to keep them low. A high-inflation regime, on the contrary, creates uncertainty, since the two policy-makers types respond differently to the disinflationary dilemma, while the public does not know in advance which policy-maker will actually be in office.

Although Ball (1992) is the most frequently quoted paper on this topic, many other explanations for the inflation-uncertainty relationship have been suggested in the literature. Departing from the Barro and Gordon (1983) game-theoretic approach[3] and inside a "new-Keynesian" perspective, Ball, Mankiw, and Romer (1988) argue that high inflation reduces nominal rigidity, thus steepening the short-run Phillips curve. This, in turn, implies an increase in inflation variability since the inflation rate becomes more sensitive to fluctuations in aggregate demand. While Ball (1992) emphasizes the role of monetary policy uncertainty, some authors point out how uncertainty about the parameters of the inflation process can give rise to a link between inflation and uncertainty. As shown in Evans and Wachtel (1993) and Holland (1993), if inflation alternates randomly between a stationary and a non-stationary regime, producing uncertainty about its persistence parameter, the conditional variance of inflation is positively related to the lagged inflation rate.

On the whole, the existence of a causal link from the inflation rate to inflation volatility has received a substantial support from the empirical work.[4]

[3] The literature includes other contributions relying on a strategic interaction between the monetary authority and the public which motivate a positive correlation between the inflation rate and its variability. However, differently from the specification underlying Equation (1), these papers generate a reverse relationship, i.e. a causal link from inflation uncertainty to the inflation rate: see Devereux (1989) and Cukierman and Meltzer (1986).

[4] The empirical evidence appears relatively stronger in cross-country studies than in time series analyses for individual countries (see, among others, Davis and Kanago (2000), who also provide a comprehensive list of previous empirical work in this area). Focusing on the time series evidence, which is clearly more relevant to our specification of the inflation process, the results are generally quite sensitive to the proxies used to measure inflation uncertainty and to the methodology of empirical investigation. Padovano (1996) criticizes the uncertainty proxies usually employed in applied research (non-overlapping moving standard deviation of the inflation rate; dispersion of inflation forecasts from survey data), since they are essentially ad-hoc, arbitrary measures which do not provide a parametric specification of the variability of the inflationary process over time. The class of Arch-models overcomes these shortcomings, allowing for a time-varying conditional variance of error terms: these models are therefore particularly suitable to explore the inflation-uncertainty link. Using a Garch approach, which allows a great flexibility to model the persistence in inflation variance, Padovano (1996) documents a significant influence of the inflation rate on inflation variability for a large sample of countries. Quite interestingly, this relationship holds both for low and for high-inflation countries. Further strong evidence of a causal link from inflation to inflation uncertainty is

In sharp contrast with Strobel (2001) (where $\pi(t)$ is treated as an exogenous variable), Equation (1) highlights a wide set of macroeconomic influences affecting the inflation rate. The main advantage of this approach is that it focuses on monetary policy as the policy-maker's control variable. In this setting, the optimal monetary policy is crucially affected not only by the degree of policy-maker's inflation aversion, but also by the degree of price stickiness in the economy. This latter influence is usually disregarded in the current literature.

We now turn to the policy-maker's incentives. The policy-maker is assumed to control money supply growth and to minimize the expected value of a loss function which is quadratic in the inflation rate and linear in surprise inflation.[5] For each period t, the loss function is thus given by:

$$L(t) = \frac{1}{2}\pi(t)^2 - b(\pi(t) - \pi^e(t)), \tag{2}$$

where $\pi^e(t)$ denotes the expected inflation rate and b is a constant positive parameter reflecting the weight assigned by the policy-maker to unanticipated inflation.

The above specification is quite standard in the literature, and closely follows that usually employed in reputational models of monetary policy (Barro and Gordon; 1983, and Barro, 1986). The first quadratic term in Equation (2) refers to inflation costs, which are assumed to rise at an increasing rate with the realized inflation rate. The latter linear term captures instead the benefits associated to unanticipated inflation. Given an expectational Phillips curve framework, an unanticipated monetary expansion leads to an increase in real economic activity, temporarily lowering the unemployment rate below its natural level.[6]

In line with the theoretical literature on political business cycles, b is a fixed parameter in Equation (2), whereas in Strobel (2001) this coefficient is assumed to follow a geometric Brownian motion. The approach taken by this author does not have, in our opinion, sound theoretical motivations. In a long run perspective, one can obviously envisage some factors that could alter the policy-maker's preference towards surprise inflation: a change in the natural level of unemployment, or an exceptional event (such as a war) producing a boom in government expenditures or a sharp rise in the outstanding real stock of nominally denominated public debt. However, since the above factors do not smoothly affect the value of b at each point in time, it seems rather implausible to model this parameter as a geometric Brownian motion. Moreover, since

provided in Holland (1995), applying Granger causality tests on postwar US data.

[5] In the context of the present model, the policy-maker can thus easily be identified with the Central Bank.

[6] The benefits from surprise inflation arise when the policy-maker views the natural rate of unemployment as excessive due to some distortion in the economy. This distortion may occur for various reasons (income taxation, unemployment benefits and the like). Other sources of benefits from surprise inflation may be associated with the proceeds from inflationary finance, lowering the real value of cash holdings and of the nominally denominated interest-bearing public debt.

Strobel (2001) includes a drift in the above process, the expected value of b is continuously increasing (or decreasing) over time.[7]

As remarked at the beginning of this section, this theoretical framework can easily be adapted to characterize the inflationary process and policy-maker's incentives inside a currency union.

In line with Equation (1), the inflation rate is assumed to be driven by the monetary policy implemented by the union's Central Bank. The dynamics of the inflationary process may thus be expressed as:

$$d\hat{\pi} = k(\hat{m} - \hat{\pi})\,dt + \sigma\left(\hat{m} + \hat{\phi}\right)dW, \tag{3}$$

where all the symbols retain their previous meanings and a hat over a variable denotes its value in the case of a monetary union between n countries.

Equation (3) can be derived as a weighted mean of n inflation rates given in Equation (1) for each country. If the weights $\alpha_i > 0$, $\forall i \in \{1, ..., n\}$ do not change over time then we can write

$$\hat{\pi} = \sum_{i=1}^{n} \alpha_i \pi_i,$$

and so

$$\begin{aligned} d\hat{\pi} &= \sum_{i=1}^{n} \alpha_i d\pi_i \\ &= k\left(\sum_{i=1}^{n} \alpha_i m_i - \sum_{i=1}^{n} \alpha_i \pi_i\right) dt + \sigma\left(\sum_{i=1}^{n} \alpha_i m_i + \sum_{i=1}^{n} \alpha_i \phi_i\right) dW \\ &\equiv k(\hat{m} - \hat{\pi})\,dt + \sigma\left(\hat{m} + \hat{\phi}\right)dW. \end{aligned}$$

Accordingly, the specification outlined in Equation (3) sets a common risk source for inflation variability inside the union; moreover, the degree of price stickiness (k) and the sensitivity of inflation variability to monetary policy (σ) are assumed to be identical for all countries. Although the above assumptions are primarily intended to yield handleable formal results, they represent a realistic approximation on purely economic grounds, since the countries forming a currency union should a priori exhibit a substantial degree of similarity in their economic structures. Note, finally, that our specification of the inflation process allows for different ϕ_i values across countries and, thus, for different inflation

[7] Modelling b as a fixed parameter is a standard assumption in most reputational models of monetary policy (Barro, 1986; Giavazzi and Pagano, 1988; Alesina and Grilli, 1992). An identical approach is taken in all "partisan" political business cycles models, where two policy-makers, with different attitudes towards inflation, alternate in office (see, among others, Alesina and Roubini, 1990). One relevant exception to the above literature is Barro and Gordon (1983), where b is treated as a time-varying parameter. Differently from Strobel (2001), however, these authors assume that b is distributed randomly with a fixed mean (therefore not evolving in an explosive fashion).

volatilities. We are therefore ascribing a certain component of inflation variability to country-specific factors. Intuitively, this assumption is meant to capture possible discrepancies in inflation variability arising from different variances of real and monetary shocks hitting the countries inside the union.[8]

Following a standard approach in the literature (Alesina and Grilli, 1992, 1993), the loss function has the same structure of those of the individual participating countries. The instantaneous loss function for the monetary union case is thus specified as:

$$\hat{L}(t) = \frac{1}{2}\hat{\pi}(t)^2 - \hat{b}\left(\hat{\pi}(t) - \hat{\pi}^e(t)\right), \quad (4)$$

where, in line with the previous specification in Equation (2), $\hat{b}$ is a time-invariant parameter reflecting the weight assigned to inflationary surprises by the union's Central Bank.

Although in the context of our model b and $\hat{b}$ are exogenous parameters, the theoretical literature on monetary integration suggests a relevant causal relationship from the former to the latter. More specifically, the set of $\{b_i\}$ coefficients, $\forall i \in \{1, ..., n\}$, defines a unique $\hat{b}$ parameter expressing the preferences of the union's Central Bank. We underline that $\hat{b}$ can be considered as a weighted mean of the values b_i but the weights are not necessarily the same as those used for computing the mean inflation rate. In particular

$$\hat{b} = \sum_{i=1}^{n} \beta_i b_i.$$

As formally shown in Alesina and Grilli (1993), $\hat{b}$ must belong to the (non empty) set of coefficients characterizing a "feasible" currency union, in which all n participating countries are not worse off with the union than without it.[9] If the monetary union is "feasible", as implicitly assumed in Equation (3), the $\hat{b}$ parameter can be thought as selected by a Council of national Central Bank governors under a majority voting rule. This parameter will therefore reflect the preferences of the "median" council member (Alesina and Grilli, 1992).[10]

[8]Devereux (1989) emphasizes the variance of real disturbances as a key factor influencing the degree of inflation variability. Cukierman and Meltzer (1986), on the other hand, stress the variance of monetary shocks. These latter might arise, in a currency union, from the demand side, i.e. from random shifts in national money demand functions.

[9]Note that a monetary union between n countries might not always be "feasible". In other words, given some configurations of individual countries preferences $\{b_i\}$, the range of parameter values at which the union is feasible might be empty. Alesina and Grilli (1993) (pp. 155-161), provide a formal discussion on this matter.

[10]This selection process for $\hat{b}$ outlines the case of a monetary union between n countries which still retain strong national and political identities. In other words, it describes how a common monetary policy is inplemented in absence of a complete political union. As discussed in Alesina and Grilli (1992), under complete political and economic integration $\hat{b}$ would instead be selected, through a majority voting rule, by all the citizens belonging to the union.

3 Optimal monetary policy

The problem is to minimize the expected value of the loss function (2), over a given time horizon, and given the dynamic equation of the inflation rate. For an infinite horizon, this dynamic problem can be written as follows:

$$\begin{cases} \min_{m_t} \mathbb{E}_{t_0} \left[\int_{t_0}^{\infty} e^{-r(t-t_0)} \left(\frac{1}{2}\pi(t)^2 - b(\pi(t) - \pi^e(t)) \right) dt \right] \\ d\pi(t) = k(m(t) - \pi(t))\, dt + \sigma(m(t) + \phi)\, dW(t), \\ \pi(t_0) = \pi_0, \end{cases} \tag{5}$$

where r is the (constant) discount rate.

From this problem we obtain the following result.

Proposition 1 *Given Problem (5) the optimal money growth rate is given by:*

$$m(t)^* = (b - \pi(t)) \frac{k}{\sigma^2} + \frac{(b+\phi) k^2}{r\sigma^2 + k^2 + k\sigma^2} - \phi. \tag{6}$$

Proof. See Appendix A ■

The role of the various parameters in the above expression has a straightforward economic interpretation.

Consider, first, the parameter b, reflecting the relative weight assigned by the policy-maker to real output growth. In line with reputational models of monetary policy, an increase in b leads to a higher money growth rate and thus to a higher long-run inflation rate.

An increase in r, i.e. the policy-maker becoming more short-sighted, determines instead a lower m^*. The reason is that, in the context of the present model, an increase in the money growth rate produces an instantaneous increase in inflation volatility, whereas the temporary benefits associated to real output expansion are only gradually reaped over time. A short-sighted policy-maker will therefore give more weight to the short-run losses induced by inflation volatility, selecting a more restrictive monetary policy.

Comparative static exercises on the above equation reveal that both the influence of k and that of σ crucially depend on a given threshold for the inflation rate. In particular we have:

$$\begin{aligned} \frac{\partial m^*}{\partial k} &\gtreqless 0 \Longleftrightarrow \pi(t) \lesseqgtr b + (b+\phi)\sigma^4 k \frac{2r + k}{(r\sigma^2 + k^2 + k\sigma^2)^2} \equiv T_k, \\ \frac{\partial m^*}{\partial \sigma} &\gtreqless 0 \Longleftrightarrow \pi(t) \gtreqless b + (b+\phi)\sigma^4 k \frac{r + k}{(r\sigma^2 + k^2 + k\sigma^2)^2} \equiv T_\sigma. \end{aligned}$$

As regards k (the sensitivity of the inflation rate to monetary policy), an increase in this parameter lowers m^* whenever the inflation rate is above a given threshold (T_k), whereas the converse holds whenever the inflation rate is under this threshold. The economic intuition behind these opposite effects becomes clear once it is realized that, given an exogenous perturbation in k, policy-maker's incentives are different according to the economy being in a "high" or in a "low" inflation regime. If prices become more flexible and the inflation rate is relatively high, m^* decreases since the government is induced to exploit this effect to moderate the inflation rate. The converse is true under a low-inflation regime, i.e. whenever the need to boost the real economy becomes relatively more important. Since a higher k decreases the real effects of monetary policy, the optimal response is now a more expansionary monetary policy (higher m^*).

Focusing on σ (the sensitivity of inflation volatility to monetary policy), comparative static analysis shows that a rise in this parameter increases (decreases) m^* whenever the inflation rate is above (under) a given inflation rate threshold (T_σ). Although these effects look rather counter-intuitive, their rationale lies in the positive relationship between inflation and inflation volatility and in the net benefit associated to a change in m^* under a high or a low inflation regime.

Assume a rise in σ in the former regime where, according to our model, inflation volatility is relatively high. In this case, a rise in m^* is, somewhat paradoxically, optimal since the marginal loss associated to a further increase in inflation volatility is lower than the marginal benefit stemming from unexpected inflation. Instead, when the inflation rate is low, then an increase in σ leads to a decrease in m since the policy-maker decides to maintain the inflation volatility to a low level. In fact, in this case, the marginal gain associated with inflation reduction is higher than the marginal gain in creating an unexpected inflation.

3.1 The optimal inflation rate

After substituting the optimal value of m into the inflation differential equation we have the following behaviour for the optimal inflation rate:

$$d\pi^* = (\mu_0 + \mu_1 \pi^*)\, dt + (\sigma_0 + \sigma_1 \pi^*)\, dW,$$

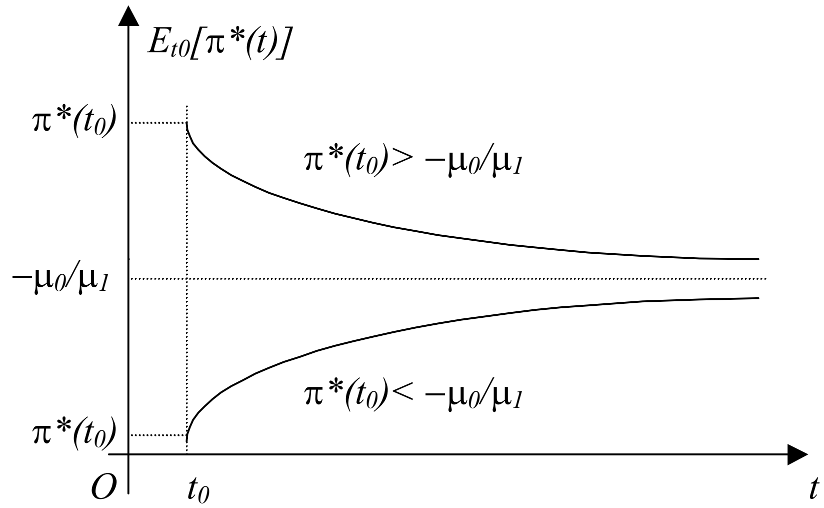
where

$$\begin{aligned}
\mu_0 &\equiv b\frac{k^2}{\sigma^2} - k\phi + \frac{(b+\phi)\, k^3}{r\sigma^2 + k^2 + k\sigma^2}, \\
\mu_1 &\equiv -k - \frac{k^2}{\sigma^2}, \\
\sigma_0 &\equiv b\frac{k}{\sigma} + \sigma\frac{(b+\phi)\, k^2}{r\sigma^2 + k^2 + k\sigma^2}, \\
\sigma_1 &\equiv -\frac{k}{\sigma}.
\end{aligned}$$

The solution for this kind of stochastic differential equation is well known in the literature (see, e.g. Øksendal, 2000) and has the following form:

$$\begin{aligned}
\pi^*(t) &= -\frac{\sigma_0}{\sigma_1} + \left(\frac{\sigma_0}{\sigma_1} + \pi^*(t_0)\right) e^{\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)(t-t_0) + \sigma_1\left(W_t - W_{t_0}\right)} \\
&\quad + \left(\mu_0 - \mu_1\frac{\sigma_0}{\sigma_1}\right)\int_{t_0}^{t} e^{\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)(t-s) + \sigma_1(W_t - W_s)} ds,
\end{aligned}$$

Figure 1: The behaviour of the optimal inflation rate



whose expected value is

$$\mathbb{E}_{t_0}\left[\pi^*(t)\right] = -\frac{\mu_0}{\mu_1} + \left(\frac{\mu_0}{\mu_1} + \pi^*(t_0)\right) e^{\mu_1 (t-t_0)},$$

and, since $\mu_1 < 0$, when t tends to infinity this expected value converges to

$$\lim_{t\to\infty} \mathbb{E}_{t_0}\left[\pi^*(t)\right] = -\frac{\mu_0}{\mu_1}. \tag{7}$$

This convergence is always monotonic and it depends on the sign of the coefficient of the exponential. In particular, the smaller the initial value of inflation in $\pi^*(t_0)$ the higher the probability that a country reaches its expected value inflation (7) by following an increasing inflation path. This behaviour is represented in Figure 1.

It is interesting to investigate the behaviour of the expected inflation rate for extreme values of the mean reverting parameter (k). In particular, we have

$$\lim_{k\to 0} \mathbb{E}_{t_0}\left[\pi^*(t)\right] = \pi^*(t_0),$$

In this case, monetary policy becomes totally ineffective and expected inflation coincides with the current inflation rate.

Conversely, when k tends to infinity we have

$$\lim_{k\to\infty} \mathbb{E}_{t_0}\left[\pi^*(t)\right] = b,$$

revealing that, when monetary policy is very effective, the inflation rate depends only on the preference parameter of the policy-maker.

As argued before, in the monetary union case k and σ are kept unchanged, whereas the other parameters are computed as weighted means. Accordingly, μ_1 and σ_1 are unaffected, while the other parameters change as follows:

$$\hat{\mu}_0 \equiv \hat{b}\frac{k^2}{\sigma^2} - k\hat{\phi} + \frac{\left(\hat{b}+\hat{\phi}\right)k^3}{r\sigma^2 + k^2 + k\sigma^2},$$

$$\hat{\sigma}_0 \equiv \hat{b}\frac{k}{\sigma} + \sigma\frac{\left(\hat{b}+\hat{\phi}\right)k^2}{r\sigma^2 + k^2 + k\sigma^2}.$$

If the weights assigned to the single countries preference parameters b_i (to compute $\hat{b}$) are identical to those to compute the union's mean inflation rate, then it is easy to check that

$$\hat{\mu}_0 = \sum_{i=1}^{n} \alpha_i \mu_{0i}, \qquad \hat{\sigma}_0 = \sum_{i=1}^{n} \alpha_i \sigma_{0i}.$$

This hypothesis will be used in the next section and we write it as follows.

Hypothesis 1 *The policy-maker of the union has a preference parameter $\hat{b}$ which is given by*

$$\hat{b} = \sum_{i=1}^{n} \alpha_i b_i.$$

As already underlined in section 2, the above hypothesis does not necessarily represent the most realistic description of the effective working of a monetary union.[11]

This taken into account, in the following section we will also discuss how our theoretical results are affected whenever hypothesis 1 does not hold.

Finally, the processes driving the optimal inflation rate of each country (π^*) and that of the monetary union ($\hat{\pi}^*$) can be written as the following vector stochastic differential equation:

$$\begin{bmatrix} d\pi^* \\ d\hat{\pi}^* \end{bmatrix} = \begin{bmatrix} \mu_0 + \mu_1\pi^* \\ \hat{\mu}_0 + \mu_1\hat{\pi}^* \end{bmatrix} dt + \begin{bmatrix} \sigma_0 + \sigma_1\pi^* \\ \hat{\sigma}_0 + \sigma_1\hat{\pi}^* \end{bmatrix} dW. \tag{8}$$

[11] According to Hypothesis 1, the "economic" weight of each country is equal to the "political" weight used to compute the preferences of the union's Central Bank. The effective working of a monetary union, however, might more or less significantly depart from the above characterization. For instance, as discussed in Alesina-Grilli (1992), the voting procedures of the ECB board are designed to met the principle "one-person one-vote" (which in the context of the present model would correspond to an identical beta for all participating countries). The inclusion of additional members (beside national Central Bank Governors) in the ECB board, however, tempers the above principle, making the voting procedures more consistent with the relative size of EMU members. See Alesina-Grilli (1992), section 5, for a more detailed discussion of this issue.

4 The optimal time for leaving a union

The purpose of this section is to assess if there exist an optimal time for leaving a union for a single country which is already participating to this monetary arrangement. We assume that this choice involves a cost (θ), proportional to the (positive) difference between home inflation and the union's inflation rate. This cost can be different for each country and it is suffered only once, at the moment of leaving the union. The assumption that a high-inflation country has a positive exit cost is quite reasonable since, while inside the union, its higher inflation is hidden beside a common inflation rate. We stress, however, that θ does not simply correspond to a monetary penalty but is more properly to be interpreted as a political-reputational cost.

If τ denotes the moment in which the union is left, the problem of each country can be formalized as follows:

$$\min_{\tau} e^{rt_0}\mathbb{E}_{t_0}\left[\int_{t_0}^{\tau} e^{-rs}\hat{L}^*(s)\,ds + e^{-r\tau}\theta\left(\pi^*(\tau)-\hat{\pi}^*(\tau)\right) + \int_{\tau}^{\infty} e^{-rs}L^*(s)\,ds\right].$$

As shown in the above expression, quitting a monetary union involves three cost components: the first is the loss while staying inside the union, the second is a once and for all exit cost, and the third is the loss after this monetary arrangement has been left.

Now, we take into account the transformation

$$\int_{\tau}^{\infty} e^{-rs}L^*(s)\,ds = \int_{t_0}^{\infty} e^{-rs}L^*(s)\,ds - \int_{t_0}^{\tau} e^{-rs}L^*(s)\,ds,$$

allowing us to rewrite the minimization problem in the following way:

$$\min_{\tau}\mathbb{E}_{t_0}\left[\int_{t_0}^{\tau} e^{-rs}\left(\hat{L}^*(s)-L^*(s)\right)ds \right.$$
$$\left. + e^{-r\tau}\theta\left(\pi^*(\tau)-\hat{\pi}^*(\tau)\right) + \int_{t_0}^{\infty} e^{-rs}L^*(s)\,ds\right],$$

and, since the last term does not depend on the control variable τ:

$$\max_{\tau}\mathbb{E}_{t_0}\left[\int_{t_0}^{\tau} e^{-rs}\left(L^*(s)-\hat{L}^*(s)\right)ds - e^{-r\tau}\theta\left(\pi^*(\tau)-\hat{\pi}^*(\tau)\right)\right].$$

We recall that the optimal loss function is given by:

$$L^*(t) = \frac{1}{2}\pi^*(t)^2 - b\left(\pi^*(t)-\pi^e(t)\right),$$

and, since the rational expectation hypothesis allows us to write $\pi^*(t) = \pi^e(t)$, we have:

$$L^*(t) = \frac{1}{2}\pi^*(t)^2.$$

The optimal stopping time problem can finally be written in the following way:

$$\max_{\tau} \mathbb{E}_{t_0} \left[\int_{t_0}^{\tau} e^{-rs} \frac{1}{2} \left(\pi^* (s)^2 - \hat{\pi}^* (s)^2 \right) ds - e^{-r\tau} \theta \left(\pi^* (\tau) - \hat{\pi}^* (\tau) \right) \right], \quad (9)$$

where the state variables of this problem are the domestic inflation and the mean inflation of the union following the stochastic differential equations in (8).

The first step for solving this problem is to check if there exist some parameters values for which the monetary union has infinite length. At this purpose, the following proposition can be derived.

Proposition 2 *Given the optimal inflation rates for each country and for the monetary union (see Equation. (8)), if:*

$$\frac{1}{2} \left(\pi^{*2} - \hat{\pi}^{*2} \right) + \theta \left(r - \mu_1 \right) \left(\pi^* - \hat{\pi}^* \right) - \theta \left(\mu_0 - \hat{\mu}_0 \right) > 0, \qquad \forall \pi^*, \hat{\pi}^* \in \mathbb{R}^+ \quad (10)$$

then there does not exist any optimal stopping time for Problem (9).

Proof. See Appendix B. ■

Proposition 2 indicates for which values of domestic and foreign inflation rates the monetary union has an infinite length. This result is clearly favourable to the long run sustainability of a monetary union and, as outlined in Equation (10), the θ parameter has a key role in this regard. As remarked before, θ is essentially to be interpreted as a political-reputational cost. As such, θ is strictly exogenous with respect to the present model and cannot artificially be manipulated by the union's central bank in order to reach the desirable parameters configuration outlined above.

When leaving the union is not costly at all for all countries ($\theta i = 0 \ \forall i \in \{1, ..., n\}$), the monetary union will immediately disintegrate.[12]

In this case, the above inequality reduces to:

$$\pi^* > \hat{\pi}^*$$

A given country will not abandon a monetary union if its own inflation rate is higher than the union's mean rate. Unfortunately, however, the above rule cannot be generalized. When each country has a zero exit cost, the above inequality cannot obviously hold for all countries.

If Equation (10) does not hold, then we can conclude what stated in the following proposition.

[12] This is a standard result in the optimal stopping literature, where a positive exit cost is crucial to avoid an exit option being immediately exercised. See, for instance, Strobel (2001), pp. 393-96.

Proposition 3 *Given the optimal inflation rates for each country and for the monetary union (see Equation. (8)), if the Condition (10) does not hold, and if r tends to zero, then one country stays in the union while its inflation rate is higher than*

$$\pi^*_{\min} = \theta\mu_1 + \sqrt{\theta^2\mu_1^2 + 2\theta\mu_0}, \tag{11}$$

and the mean inflation rate is lower than

$$\hat{\pi}^*_{\max} = \theta\mu_1 + \sqrt{\theta^2\mu_1^2 + 2\theta\hat{\mu}_0}. \tag{12}$$

Proof. See Appendix B ■

The result stated in Proposition 3 identifies two critical thresholds: (i) one for domestic inflation, and (ii) one for the union's mean inflation rate. The former threshold represents a lower limit. When the domestic inflation goes down this limit, it is optimal for the single country to leave the union since it will benefit from a low inflation rate. The latter threshold represents instead an upper limit. When the union's inflation rate is higher than this threshold, the single country leaves the union because, in the future, it will be able to benefit from a relative low inflation rate.

In other words, a country leaves a monetary union either when it is too "virtuous" with respect to the other members or when the union as a whole is excessively prone towards inflation.

Now, we want to start our analysis in a case where the monetary union already exists. This means that the initial values π^*_0 and $\hat{\pi}^*_0$ must satisfy:

$$\pi^*_0 > \pi^*_{\min}, \qquad \hat{\pi}^*_0 < \hat{\pi}^*_{\max}.$$

From the first inequality we can derive a necessary condition for the second one to hold. In fact, after writing the condition for country i:

$$\pi^*_{0i} > \theta_i\mu_1 + \sqrt{\theta_i^2\mu_1^2 + 2\theta_i\mu_{0i}},$$

we can write:

$$\sum_{i=1}^{n} \alpha_i\pi^*_{0i} > \sum_{i=1}^{n} \alpha_i\theta_i\mu_1 + \sum_{i=1}^{n} \alpha_i\sqrt{\theta_i^2\mu_1^2 + 2\theta_i\mu_{0i}},$$

which implies[13]

$$\hat{\pi}^*_0 > \hat{\theta}\mu_1 + \sum_{i=1}^{n} \alpha_i\sqrt{\theta_i^2\mu_1^2 + 2\theta_i\mu_{0i}}.$$

Since we want the condition $\hat{\pi}^*_0 < \hat{\pi}^*_{\max}$ to hold, then it must be true that

$$\hat{\theta}\mu_1 + \sum_{i=1}^{n} \alpha_i\sqrt{\theta^2\mu_1^2 + 2\theta\mu_{0i}} < \hat{\pi}^*_{\max}.$$

[13] We recall $\sum_{i=1}^{n} \alpha_i = 1$.

Under the simple hypotheses[14]

$$\begin{aligned} \theta_i &= \theta, \qquad \forall i = 1, ..., n \\ \alpha_i &= \beta_i, \qquad \forall i = 1, ..., n \end{aligned}$$

then we can write

$$\sum_{i=1}^{n} \alpha_i \sqrt{\frac{1}{2}\theta\mu_1^2 + \mu_{0i}} < \sqrt{\frac{1}{2}\theta\mu_1^2 + \sum_{i=1}^{n} \alpha_i \mu_{0i}},$$

which is clearly true. This means that there always exists some place for the union (at least) to begin.

We can compute the expected time in which a single country will leave the monetary union by solving the following equations:

$$\mathbb{E}_{t_0}\left[\pi^*(t)\right] < \pi^*_{\min}, \qquad \mathbb{E}_{t_0}\left[\hat{\pi}^*(t)\right] > \hat{\pi}^*_{\max},$$

which can be written as follows:

$$\begin{aligned} \left(\mu_0 + \mu_1 \pi_0^*\right) e^{\mu_1 (t - t_0)} &> \mu_0 + \mu_1 \pi^*_{\min}, \\ \left(\hat{\mu}_0 + \mu_1 \hat{\pi}_0^*\right) e^{\mu_1 (t - t_0)} &< \hat{\mu}_0 + \mu_1 \hat{\pi}^*_{\max}. \end{aligned}$$

Now, we have to distinguish four different conditions on this two inequalities:

1. if $\mu_0 + \mu_1 \pi_0^* < 0 \Leftrightarrow \pi_0^* > -\frac{\mu_0}{\mu_1}$, the first inequality never holds and the expected value of π^* never falls down the level $\pi^*_{\min}$;

2. if $\pi^*_{\min} < \pi_0^* < -\frac{\mu_0}{\mu_1}$, the solution to the first inequality is

$$t < t_0 + \frac{1}{\mu_1} \ln \frac{\mu_0 + \mu_1 \pi^*_{\min}}{\mu_0 + \mu_1 \pi_0^*},$$

but, since the expected value of π^* is monotonically increasing in t (under the condition of this point). we can conclude that, for $t \geq t_0$ the expected value of π^* is always lower than $\pi^*_{\min}$;

3. if $\hat{\mu}_0 + \mu_1 \hat{\pi}_0^* < 0 \Leftrightarrow \hat{\pi}_0^* > -\frac{\hat{\mu}_0}{\mu_1}$, the second inequality always holds and the expected value of $\hat{\pi}^*$ is always greater than $\hat{\pi}^*_{\max}$ (which is obvious since $\hat{\pi}^*_{\max}$ is always lower than $-\hat{\mu}_0/\mu_1$);

4. if $\hat{\pi}_0^* < \hat{\pi}^*_{\max} < -\frac{\hat{\mu}_0}{\mu_1}$, the solution of the second inequality is

$$t > t_0 + \frac{1}{\mu_1} \ln \frac{\hat{\mu}_0 + \mu_1 \hat{\pi}^*_{\max}}{\hat{\mu}_0 + \mu_1 \hat{\pi}_0^*}.$$

We have summarized these results in Table 1 and we can state what follows.

[14] The second hypothesis implies $\hat{\mu}_0 = \sum_{i=1}^{n} \alpha_i \mu_{0i}$.

Table 1: Optimal expcted stopping time according to the initial inflation levels

π_0^*	$\hat{\pi}_0^*$	**Optimal expected stopping time**
$> -\frac{\mu_0}{\mu_1}$	$> -\frac{\hat{\mu}_0}{\mu_1}$	$t = t_0$
$> -\frac{\mu_0}{\mu_1}$	$< \hat{\pi}^*_{\max}$	$t = t_0 + \frac{1}{\mu_1} \ln \frac{\hat{\mu}_0 + \mu_1 \hat{\pi}^*_{\max}}{\hat{\mu}_0 + \mu_1 \hat{\pi}^*_0}$
$< -\frac{\mu_0}{\mu_1}$	$> -\frac{\hat{\mu}_0}{\mu_1}$	$t = t_0$
$< -\frac{\mu_0}{\mu_1}$	$< \hat{\pi}^*_{\max}$	$t = t_0$

Proposition 4 *Given the optimal inflation rates for each country and for the monetary union (see Equation (8)), if the Condition (10) does not hold, and if r tends to zero, then the expected optimal stopping time is*

$$t = t_0 + \frac{1}{\mu_1} \ln \frac{\hat{\mu}_0 + \mu_1 \hat{\pi}^*_{\max}}{\hat{\mu}_0 + \mu_1 \hat{\pi}^*_0},$$

when $\pi_0^* > -\frac{\mu_0}{\mu_1}$ and $\hat{\pi}_0^* < \hat{\pi}^*_{\max} \left(< -\frac{\hat{\mu}_0}{\mu_1}\right)$, while it is immediate in all the other cases.

Now, we outline that if Hypothesis 1 holds, then the only case in which the end of the monetary union is not immediate is impossible. In fact, since under Hypothesis 1 both $\hat{\pi}_0^*$ and $\hat{\mu}_0$ are weighted means of π_0^* and μ_0 respectively (with the same weights), then the following passages hold for each $i \in \{1, ..., n\}$:

$$\begin{aligned} \pi^*_{0i} &> -\frac{\mu_{0i}}{\mu_1} \Rightarrow \alpha_i \pi^*_{0i} > -\alpha_i \frac{\mu_{0i}}{\mu_1} \\ &\Rightarrow \sum_{i=1}^{n} \alpha_i \pi^*_{0i} > \sum_{i=1}^{n} -\alpha_i \frac{\mu_{0i}}{\mu_1} \\ &\Rightarrow \hat{\pi}^*_0 > -\frac{\hat{\mu}_0}{\mu_1}. \end{aligned}$$

Accordingly, we can state what follows.

Corollary 1 *Given the optimal inflation rates for each country and for the monetary union (see Equation. (8)), if the Condition (10) does not hold, and if r tends to zero, then under Hypothesis 1 the expected optimal stopping time is immediate ($t^* = t_0$).*

Overall, the analysis performed in the present section shows that, under a particular condition on models's parameters, the expected lenght of a monetary union is infinite. If the above condition is not satisfied, the optimal expected stopping time is zero unless the "economic" weight of each country (α_i) is different from its "political" weight (β_i). In this latter case, the expected optimal stopping time of a monetary union takes a finite value. Whenever the condition ensuring the perpetual life of a monetary union (Equation (10)) breaks down, a departure from the symmetry condition $\alpha_i = \beta_i$ plays therefore a positive influence on its long run sustainability. We underline, at this purpose, that more or less significant departures from the symmetry condition $\alpha_i = \beta_i$ are very likely to characterize the effective working of a monetary union (see Footnote 11).

It must finally be stressed that the results stated above refer to the expected optimal stopping time, and not to the exact time at which a monetary union will eventually break down. As outlined in Equations (11) and (12), the actual stopping time can only be defined as the first time either domestic inflation goes under a lower threshold, or the union's inflation rate goes above an upper threshold.

5 Conclusion

Since earlier work in the theory of optimum currency areas, the literature on monetary integration has almost entirely focused on the prerequisites allowing a set of countries to give up their monetary sovereignty. Although the analysis of potentially destabilizing influences has received a greater attention in recent years, contributions in this area have typically been confined to the transition phase preceding the formal establishment of a currency union. The issue of its long run sustainability, on the other hand, has so far received a surprisingly scant attention; in other words, once a currency union is established, this regime shift is usually regarded as a strictly irreversible process.

Treating the introduction of a common currency as an irrevocable choice is a relevant shortcoming in the existing theoretical literature. The above assumption, moreover, stands in sharp contrast with the historical record of the last two centuries. As revealed by the empirical evidence, the actual experience of currency unions has indeed been remarkably mixed: while some of them were highly successful, others ultimately proved unsustainable.

In a recent innovative contribution, Strobel (2001) examines the value of the option of monetary disintegration inside a two-country model relying on an optimal stopping framework. The main result obtained in this paper is that the return to monetary independence is crucially affected by a trigger value in relative inflation preference parameters. Moreover, for standard configurations of parameters values, the collapse of a monetary union turns out to be a rather unlikely event.

In the present paper we retain an optimal stopping approach in order to assess the long run sustainability of a monetary union between n participating countries. Our contribution improves upon Strobel (2001) in two respects. First, we remove an apparently unrealistic assumption, namely that inflation preference parameters change continuously, in a random fashion, following a geometric Brownian motion. Second, we provide the analysis with a richer macroeconomic framework. While the inflation rate is exogenous in Strobel

(2001), we model this variable as a stochastic process whose drift and volatility depend on the money growth rate. This specification is consistent with recent theoretical and empirical developments; moreover it allows to cast the dynamic optimization problem faced by the policy-maker inside a more realistic set up, where monetary policy is the crucial control variable.

The main results achieved in this paper may be summarized as follows.

Under suitable values of some model's parameters, and if the exit cost is strictly positive, an optimal stopping time problem does not exist and the monetary union has an infinite length. If the above values of model's parameters are not satisfied, and the exit cost remains strictly positive, the optimal expected stopping time of a monetary union is significantly affected by the coefficients expressing the relative "economic" (α_i) and "political" (β_i) weights of the participating countries. Given a symmetry condition $\alpha_i = \beta_i$, the optimal expected stopping time is zero. Conversely, if the "political" weight of each country does not exactly correspond to its "economic" weight, the optimal expected stopping time takes a finite value. We underline that more or less significant departures from the symmetry condition characterize the effective working of existing monetary unions.Therefore, if exogenous shifts in the exit cost parameter would jeopardize the long run sustainability of a monetary union, asymmetries between α_i and β_i would anyway play a positive influence in this regard. While all the results outlined above are defined in terms of expected values (the optimal expected stopping time), the model also characterizes the conditions governing the effective breakdown of a monetary union. At this purpose, assuming that policy-makers are sufficiently far-sighted, we find two critical thresholds: (i) a lower level for domestic inflation, and (ii) an upper level for the union's (mean) inflation rate. The optimal stopping time for a single country to leave the union is either the first time its domestic inflation goes under the former threshold, or the first time the union's inflation exceeds the latter threshold. This result has a strong economic intuition: the monetary union will actually collapse either when a single country is too "virtuous" with respect to the other members or when the union as a whole is excessively prone towards inflation. The key influence exerted by some inflationary thresholds stands out as a relevant analogy with Strobel (2001) theoretical contribution. However, while in Strobel (2001) these thresholds are defined in terms of relative inflation preference parameters, in the present paper they are more realistically defined with reference to the actual pattern exhibited by inflation dynamics.

Is this paper too optimistic as regards the long run perspectives of a monetary union? A tentative answer to this question raises two issues which, in our opinion, are quite relevant to a critical appraisal of our results.

The former topic involves the need for an in-depth sensitivity analysis relying on the current version of this theoretical model. The threshold values of inflation rates triggering the collapse of a monetary union ultimately depend, beside the exit cost, on a large set of structural coefficients. More specifically, this set of parameters includes: the degree of price stickiness, the sensitivity of inflation volatility to monetary policy, idiosyncratic factors affecting inflation volatility, and the inflation preference coefficients (of the single countries and of the monetary union as a whole). As shown in section four, both critical thresholds are highly nonlinear functions of all these parameters. Numerical simulations of the model, based on different parameters configurations, could therefore provide interesting empirical insights, displaying the effective degree of sustainability of a currency union under alternative structural characterizations of participating countries.

A latter topic deserving further attention concerns the robustness of our results to some refinements in the underlying theoretical model. In this paper, the policy-maker's dynamic optimization problem relies on a loss function including inflation costs and a temptation to inflate in order to boost the real economy. Although this specification is quite standard in the literature, it implicitly assumes that, for all countries joining a monetary union, debt dynamics is fully under control. If this is not the case, a trade-off between inflation and debt arises: intuitively, a too tight monetary policy makes the debt service so high that the optimal choice may be a partial or total default. Jahjah (2000, 2001) formalizes these arguments adapting Calvo (1988) seminal paper to the case of a monetary union. As shown by this author, the relative size of "fiscally weak" countries is crucial to determine the equilibrium outcome; moreover, some equilibria exist in which the above countries are forced outside a monetary union. Jahjah (2000, 2001) raises a relevant theoretical issue; however, since optimal monetary and fiscal policies are derived inside a two-perod model, the long run sustainability of a monetary union cannot adequately be explored in his set up. Allowing for a default cost in the policy-maker's loss function, and extending the analysis inside an optimal stopping framework, could therefore represent a promising research strategy for future work.

A The optimal money growth rate

From Problem (5) we have the following Hamiltonian:

$$\begin{aligned}\mathcal{H} &= e^{-r(t-t_0)}\left(\frac{1}{2}\pi^2(t) - b\left(\pi(t) - \pi^e(t)\right)\right) + J_\pi k\left(m(t) - \pi(t)\right) \\ &\quad + \frac{1}{2}J_{\pi\pi}\left(m(t)+\phi\right)^2\sigma^2,\end{aligned}$$

where $J(\pi, t)$ is the value function solving the Hamilton-Jacobi-Bellman partial differential equation (HJB) and the subscripts on J indicate the partial derivatives. The first and second order conditions which must hold on this Hamiltonian are

$$\begin{aligned}\frac{\partial\mathcal{H}}{\partial m} &= J_\pi k + J_{\pi\pi}(m+\phi)\sigma^2 = 0, \\ \frac{\partial^2\mathcal{H}}{\partial m^2} &= J_{\pi\pi}\sigma^2 > 0.\end{aligned}$$

From the first order condition we obtain the optimal money growth rate as a function of $J(\pi, t)$:

$$m^* = -\frac{J_\pi}{J_{\pi\pi}} \frac{k}{\sigma^2} - \phi,$$

while the second order condition holds if the function $J(\pi, t)$ is convex in π.

When we substitute the value m^* into the Hamiltonian, we obtain the HJB equation (with its transversality condition):

$$\begin{cases} J_t + e^{-r(t-t_0)} \left(\frac{1}{2}\pi^2 - b(\pi - \pi^e)\right) - \frac{1}{2}\frac{J_\pi^2}{J_{\pi\pi}}\frac{k^2}{\sigma^2} - J_\pi k(\phi + \pi) = 0, \\ \lim_{t\to\infty} J(t, \pi) = 0. \end{cases}$$

This kind of problem is generally solved thanks to a separability condition on the value function. Here, we check for the following functional form:

$$J(t, \pi) = e^{-r(t-t_0)} f(\pi),$$

for which the transversality condition holds, and we can rewrite the HJB equation as

$$-rf + \frac{1}{2}\pi^2 - b(\pi - \pi^e) - \frac{1}{2}\frac{f_\pi^2}{f_{\pi\pi}}\frac{k^2}{\sigma^2} - f_\pi k(\phi + \pi) = 0.$$

Now, since the function $f(\pi)$ is supposed to inherit its form from the objective function, then we try the following guess function:

$$f(\pi) = A\pi^2 + B\pi + C,$$

where A, B, and C are constant parameters whose values must be determined. The previous differential equation becomes

$$\begin{aligned} 0 &= -r\left(A\pi^2 + B\pi + C\right) + \frac{1}{2}\pi^2 - b(\pi - \pi^e) - \frac{1}{4A}(2A\pi + B)^2 \frac{k^2}{\sigma^2} \\ &\quad - (2A\pi + B)k(\phi + \pi), \end{aligned}$$

and this equation can be written as a system of three equations in three unknowns in the following way:

$$\begin{cases} \left(-rA + \frac{1}{2} - A\frac{k^2}{\sigma^2} - 2kA\right)\pi^2 = 0, \\ \left(-rB - b - B\frac{k^2}{\sigma^2} - 2k\phi A - kB\right)\pi = 0, \\ -rC + b\pi^e - \frac{1}{4A}\frac{k^2}{\sigma^2}B^2 - kB\phi = 0. \end{cases}$$

This system has one and only one solution and, in particular, we have:

$$\begin{aligned} A &= \frac{\sigma^2}{2(r\sigma^2 + k^2 + 2k\sigma^2)}, \\ B &= -\frac{b\left(r\sigma^2 + k^2 + 2k\sigma^2\right) + k\phi\sigma^2}{(r\sigma^2 + k^2 + 2k\sigma^2)(r\sigma^2 + k^2 + k\sigma^2)}\sigma^2, \end{aligned}$$

where we underline that the positive value of A makes the second order condition hold.

After substituting the first and the second derivative of the found value function into the optimal money growth rate, we can write:

$$m^* = -\frac{2A\pi + B}{2A}\frac{k}{\sigma^2} - \phi,$$

and, finally:

$$m^* = (b - \pi)\frac{k}{\sigma^2} + \frac{(b + \phi)\,k^2}{r\sigma^2 + k^2 + k\sigma^2} - \phi.$$

B The optimal stopping time

Given Problem (9), its characteristic operator is

$$\mathcal{A}\left(-e^{-rs}\theta\left(\pi^*(s) - \hat{\pi}^*(s)\right)\right) + e^{-rs}\frac{1}{2}\left(\pi^*(s)^2 - \hat{\pi}^*(s)^2\right),$$

where $\mathcal{A}$ is the characteristic operator of an Itô diffusion and the stochastic variables π^* and $\hat{\pi}^*$ follow the Processes (8). It is well known (Øksendal, 2000) that in the region where the characteristic operator is positive, there does not exist any optimal stopping time. This positivity condition can be written as

$$\begin{gathered} re^{-rs}\theta\left(\pi^*(s) - \hat{\pi}^*(s)\right) - e^{-rs}\theta\left(\mu_0 + \mu_1\pi^*(s)\right) \\ +e^{-rs}\theta\left(\hat{\mu}_0 + \mu_1\hat{\pi}^*(s)\right) + e^{-rs}\frac{1}{2}\left(\pi^*(s)^2 - \hat{\pi}^*(s)^2\right) > 0, \end{gathered}$$

which can be simplified as in Proposition 2.

When this inequality does not hold, the optimal stopping time can have a finite solution which can be found by solving the following partial differential equation for the function $G\left(\pi_0^*, \hat{\pi}_0^*\right)$:

$$\begin{aligned} 0 \;\; = \;\; & -rG + \frac{\partial G}{\partial \pi^*}\left(\mu_0 + \mu_1\pi^*\right) + \frac{\partial G}{\partial \hat{\pi}^*}\left(\hat{\mu}_0 + \mu_1\hat{\pi}^*\right) \\ & +\frac{1}{2}\left(\sigma_0 + \sigma_1\pi^*\right)^2\frac{\partial^2 G}{\partial \pi^{*2}} + \left(\sigma_0 + \sigma_1\pi^*\right)\left(\hat{\sigma}_0 + \sigma_1\hat{\pi}^*\right)\frac{\partial^2 G}{\partial \pi^*\partial\hat{\pi}^*} \\ & +\frac{1}{2}\left(\hat{\sigma}_0 + \sigma_1\hat{\pi}^*\right)^2\frac{\partial^2 G}{\partial \hat{\pi}^{*2}} + \frac{1}{2}\left(\pi^{*2} - \hat{\pi}^{*2}\right), \end{aligned}$$

with the boundary condition

$$G\left(\pi_0^*, \hat{\pi}_0^*\right) = -\theta\left(\pi_0^* - \hat{\pi}_0^*\right).$$

By using a separation argument, we can state that the function solving this problem can be written as

$$G\left(\pi^*, \hat{\pi}^*\right) = f\left(\pi^*\right) + g\left(\hat{\pi}^*\right),$$

where the two functions $f(\pi^*)$ and $g(\hat{\pi}^*)$ solve the following ordinary differential equations:

$$\begin{cases} (\mu_0 + \mu_1 \pi^*) \frac{\partial f(\pi^*)}{\partial \pi^*} + \frac{1}{2} (\sigma_0 + \sigma_1 \pi^*)^2 \frac{\partial^2 f(\pi^*)}{\partial \pi^{*2}} - r f(\pi^*) + \frac{1}{2} \pi^{*2} = 0, \\ f(\pi_0^*) = -\theta \pi_0^*, \end{cases}$$

$$\begin{cases} (\hat{\mu}_0 + \mu_1 \hat{\pi}^*) \frac{\partial g(\hat{\pi}^*)}{\partial \hat{\pi}^*} + \frac{1}{2} (\hat{\sigma}_0 + \sigma_1 \hat{\pi}^*)^2 \frac{\partial^2 g(\hat{\pi}^*)}{\partial \hat{\pi}^{*2}} - r g(\hat{\pi}^*) - \frac{1}{2} \hat{\pi}^{*2} = 0, \\ g(\hat{\pi}_0^*) = \theta \hat{\pi}_0^*, \end{cases}$$

which can be written under a general form

$$\begin{cases} (\mu_0 + \mu_1 x) \frac{\partial h(x)}{\partial x} + \frac{1}{2} (\sigma_0 + \sigma_1 x)^2 \frac{\partial^2 h(x)}{\partial x^2} - r h(x) + \frac{1}{2} a x^2 = 0, \\ h(x_0) = -a \theta x_0, \end{cases}$$

where, if $a = 1$ and $x = \pi^*$ then $h(x) = f(\pi^*)$ while if $a = -1$ and $x = \hat{\pi}^*$ then $h(x) = g(\hat{\pi}^*)$.

Unfortunately, even if the solution to this kind of ODE can be expressed thanks to the Whittaker functions, in our case we need an explicit solution for computing the threshold x_0. Even if we do not know an explicit solution for $h(x)$ we are able to compute a solution when r tends to zero. In this case, in fact, we have the general solution

$$h(x) = -a\theta x_0 + \int_{x_0}^{x} \left((C_1 - a\lambda(u)) \left(\frac{u}{\sigma_0 + \sigma_1 u} \right)^2 \left(\frac{\partial \lambda(u)}{\partial u} \right)^{-1} \right) du,$$

where

$$\lambda(u) = \int \left(\frac{u}{\sigma_0 + \sigma_1 u} \right)^2 (\sigma_0 + \sigma_1 u)^{2\frac{\mu_1}{\sigma_1^2}} e^{-2\frac{-\mu_1 \sigma_0 + \sigma_1 \mu_0}{\sigma_1^2 (\sigma_0 + \sigma_1 u)}} du,$$

and C_1 is a constant whose value must be determined in order to satisfy the smooth principle

$$\left. \frac{\partial h(x)}{\partial x} \right|_{x_0} = -a\theta,$$

and so we have

$$C_1(x_0) = a\lambda(x_0) - a\theta \left(\frac{x_0}{\sigma_0 + \sigma_1 x_0} \right)^{-2} \frac{\partial \lambda(x_0)}{\partial x_0}.$$

Thus, we have found the functional form of $h(x)$, depending on x_0. The last step is to find the value of x_0 maximizing the function $h(x, x_0)$, given the value of x. We can write

$$\begin{aligned} \frac{\partial h(x, x_0)}{\partial x_0} &= -a\theta - \left((C_1(x_0) - a\lambda(x_0)) \left(\frac{x_0}{\sigma_0 + \sigma_1 x_0} \right)^2 \left(\frac{\partial \lambda(x_0)}{\partial x_0} \right)^{-1} \right) \\ &\quad + \frac{\partial C_1(x_0)}{\partial x_0} \int_{x_0}^{x} \left(\frac{u}{\sigma_0 + \sigma_1 u} \right)^2 \left(\frac{\partial \lambda(u)}{\partial u} \right)^{-1} du. \end{aligned}$$

The two first terms vanish and we have just to solve

$$\frac{\partial C_1(x_0)}{\partial x_0} = 0.$$

Let us show the main computations. We have to satisfy the following equality:

$$a\frac{\partial \lambda(x_0)}{\partial x_0} - a\theta \frac{\partial}{\partial x_0}\left(\left(\frac{x_0}{\sigma_0 + \sigma_1 x_0}\right)^{-2} \frac{\partial \lambda(x_0)}{\partial x_0}\right) = 0.$$

We recall that

$$\frac{\partial \lambda(x_0)}{\partial x_0} = \left(\frac{x_0}{\sigma_0 + \sigma_1 x_0}\right)^2 (\sigma_0 + \sigma_1 x_0)^{2\frac{\mu_1}{\sigma_1^2}} e^{-2\frac{-\mu_1\sigma_0 + \sigma_1\mu_0}{\sigma_1^2(\sigma_0 + \sigma_1 x_0)}},$$

and so

$$\begin{aligned} & \left(\frac{x_0}{\sigma_0 + \sigma_1 x_0}\right)^2 (\sigma_0 + \sigma_1 x_0)^{2\frac{\mu_1}{\sigma_1^2}} e^{-2\frac{-\mu_1\sigma_0 + \sigma_1\mu_0}{\sigma_1^2(\sigma_0 + \sigma_1 x_0)}} \\ = \; & \theta\frac{\partial}{\partial x_0}\left((\sigma_0 + \sigma_1 x_0)^{2\frac{\mu_1}{\sigma_1^2}} e^{-2\frac{-\mu_1\sigma_0 + \sigma_1\mu_0}{\sigma_1^2(\sigma_0 + \sigma_1 x_0)}}\right), \end{aligned}$$

from which we have the final result

$$x_0^{\#} = \theta\mu_1 + \sqrt{\theta^2\mu_1^2 + 2\theta\mu_0}.$$

The sign of the second derivative crucially depends on the sign of the parameter a. Some very long computations lead to the following result:

$$a \gtrless 0 \Leftrightarrow \left.\frac{\partial^2 C_1(x_0)}{\partial x_0^2}\right|_{x_0 = x_0^{\#}} \gtrless 0,$$

and so we can conclude what stated in Proposition 3.

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