Stock price dynamics: an empirical test of the chartist-fundamentalist hypothesis[1]

Several theoretical papers investigating the effects of financial markets microstructure on asset prices focus on the interaction between a group of more informed (rational) and a group of less informed (noise, liquidity, near rational) traders. In this framework we propose an empirical test of a theoretical hypothesis developed by Sethi (1996) in which fundamentalists trade on deviations between the observed and the "fundamental" price and chartists trade on the basis of their trend expectations. Empirical findings on the dynamics of S&P constituents' stock prices are broadly consistent with model predictions and highlight the predominant role of chartists' during the stock market boom of the last part of the 90's.

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Introduction

A well known result in finance is the identification of equilibria implying the coexistence of informed (rational) and uninformed (both rational and noise) traders. This is because, in a market with costly information, the effort spent by those more informed in obtaining such information has to be compensated in equilibrium by returns from transactions made with uninformed traders. Therefore the former will never find it convenient to achieve a full information equilibrium in which nobody can use market information against others (Grossman-Stiglitz, 1980).

Within this framework a recent branch studying financial markets microstructure takes into account the widely observable presence of trend following traders and focuses in particular on the relationship between fundamentalists and chartists (Goodhart, 1988; Frankel-Froot, 1990; Kirman, 1991; Pilbeam, 1995; Sethi, 1996; Franke-Sethi, 1998). In these papers the first group of traders tries to compute the "fundamental value" of the stock according to a generally established rule and buys (sells) stocks which are undervalued (overvalued) with respect to their estimated fundamental. The remaining share of traders has a different perspective and, roughly speaking, adopts trend-following strategies buying (selling) a rising (falling) stock even if it is above (below) the fundamental value predicted by the first group of traders.

The trade-off between model complexity and empirical testability exists also in this literature. Therefore, to test these theoretical hypotheses we need to choose among those models proposing the simplest theoretical setting, at the cost of oversimplifying some assumptions. A valuable benchmark for empirical testing is represented by Sethi (1996) which proposes a two equation dynamic system in which variations of stock prices and of their trends depend on the demands of fundamentalists and chartists looking respectively at deviations from the fundamental and trend dynamics.

The paper proposes an empirical test of this model and is divided into six sections (including introduction and conclusions). In the second section we briefly summarize the Sethi (1996) model to provide theoretical underpinnings to the hypotheses which will be tested in the empirical analysis. The third and the fourth sections tackle some crucial methodological issues in the definition, respectively, of the fundamental and of the price trend explaining why, in the first case, a discounted cash flow approach is adopted. The fifth section presents and comments empirical findings obtained by estimating the two equation system.

2. The theoretical model and the specification for empirical testing

In the Sethi (1996) model there is a single asset in fixed supply. p is defined as the log of the price of the asset. Chartists identify a trend p in price movements. Their demand is: $D^c(p) = mg(p)$ (1)

where $m \in [0,1]$ is their share among total traders and g expresses their reaction to the observed trend.

Fundamentalists buy the stock when its price p is below what they believe to be the fundamental value (f) of the asset.[2] By assuming $f = 0$ for simplicity, p coincides with the deviation from the fundamental. The demand of fundamentalists may therefore be written as: $D^f(p) = -(1-m)qp$ (2)

where q is their sensitivity to deviations of the observed from the fundamental value and $(1-m)$ their share of total wealth.

Aggregate demand is therefore the sum of (1) and (2). As a consequence:

$$D(p,p) = mg(p) - (1-m)qp \qquad (3)$$

The model does not assume continuous market clearing following in this respect Beja and Goldman (1980) and Chiarella (1992). The assumption that not all trades are instantaneously executed is obviously prone to criticism. We slightly depart from this original approach since we need to specify the dynamics of the model in discrete and not in continuous time for estimation purposes. This is convenient since the impact of excess demand on price changes in a sufficiently large time interval does not

[2] The same definition is adopted by Beja-Goldman (1980) and Frankel and Froot (1986)

require the rejection of the assumption of instantaneous market clearing. What we just say is that price movements in a given time interval depend on the relative volume of orders of buyers and sellers.

Price law of motion in the model is:

$$\frac{\partial p}{\partial t} = bD(p;p) \quad (4)$$

or, taking in account the explicit form of the demand,

$$\frac{\partial p}{\partial t} = b\left[mg(p) - (1-m)qp\right] \quad (5)$$

Using the Euler discrete approximation P_t to the solution $p(t)$[3] it is possible to rewrite (5) as:

$$P_t = (1 + b\Delta t)P_{t-1} \quad (6).$$

Therefore our discrete time price adjustment equation becomes, for $\Delta t = 1$, in our set of time intervals:

$$p_t - p_{t-1} = \Delta p = b\left[mg(p_{t-1}) - (1-m)qp_{t-1}\right] \quad (7)$$

where b measures the speed of adjustment under the excess demand pressure.

Chartists have adaptive expectations based on the difference between current and expected price changes:

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[3] The numerical solution of a differential equation should inherit (in a suitable discrete sense), the important stability properties of the continuous problem. We consider the explicit and implicit Euler methods for a prototypical dissipative problem:

$\frac{\partial y}{\partial t} = -ly$, $t \geq 0, l > 0, y(0) = y_0$, the solution is $y(t) = y_0 e^{-lt}$ and since $l > 0$ all solutions tend to 0 as $t \to \infty$. For a fixed time step Δt, let $t_0 = 0$ and $t_n = t_{n-1} + \Delta t$ for $n \geq 1$. The discrete approximation Y_n, using explicit Euler scheme, to $y(t_n)$ is given by: $Y_n = (1 - l\Delta t)Y_{n-1}$. (Hirsch and Smale, 1974)

$$\frac{\partial p}{\partial t} = a\,(\frac{\partial p}{\partial t} - p) \qquad (8)$$

where a measures the extent of the error correction. Also in this case, after discretising and substituting for (7), the (8) could be written as:

$$\Delta p = a\,(mg(p_{t-1}) - (1-m)qp_{t-1} - p_{t-1}) \qquad (9)$$

with parameters $a, b, q > 0$.

Equations (7) and (9) represent the discrete-time form of the two-dimensional differential equation system (5) and (8), which is subject to our empirical test. [4]

To estimate the system composed by (7) and (9) we need to specify the $g(p_t)$ function. In this respect we assume, following Sethi, that the responsiveness of chartists' demand to the perceived trend must be bounded. It is in fact reasonable to believe that the positive marginal effect on their demand of a further increase (decrease) in the perceived trend must become null for very high (low) values of the trend itself. The chosen functional form which satisfies this property is $g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$.Alternatively, we also take in account the functional

[4] The system of differential equations (5) and (8) is dynamically stable only when $\frac{\partial p}{\partial t} = \frac{\partial p}{\partial t} = 0$ and μ ≠1. Under these conditions stock price is equal to the fundamental and chartists (whose share of total market wealth is lower than one) expect that prices do not change. The system is locally stable if price adjustment to excess demand and chartists' response to perceived trend changes is small ($bg'(0) \leq 1$). If, on the contrary, $bg'(0) > 1$, the system may be locally stable or locally unstable depending on the bifurcation value of μ. The intuition is that, if the share of the wealth in the hands of chartists' is higher than a certain threshold, their trend following behaviour will make the system unstable.

form $g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$, chosen by Sethi (1996) for simulation of the model in the continuous time.

We study the behavior of the two kinds of financial market participants under the hypothesis of an exogenous wealth distribution between chartists and fundamentalists. $m \in [0,1)$ is therefore assumed as exogenous.[5] We then use the discrete approximation of the system (5)-(8), represented by (7)-(9), to estimate the two agents reaction functions and we verify if the coefficients found are compatible with stability conditions of the equilibrium in the continuous case.

Finally, we remove the simplifying assumption that $f = 0$,[6] so that the system of equations (7) and (9) may be estimated only once we properly define the fundamental and the trend followed by chartists. The following sections will illustrate our methodological choices on these two crucial variables.

3.1 The construction of the fundamental using the DCF approach.

Our DCF approach is based on I/B/E/S forecasts and has the advantage of using current net earnings as the only accounting variable.[7] This approach

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[5] The stability of the equilibrium of the system (5)-(8), ensured only for a limited range of values of the wealth held by chartists, is analysed in the following sections.
6 In this case it is necessary to read $p - f$ in the place of p in the system (7)-(9) that we empirically test.
[7] Accounting and economic literature usually adopt at least three different approaches to calculate the fundamental value of a stock: i) the comparison of balance sheet multiples (EBITDA, EBIT) for firms in the same sector; ii) the residual income method; iii) the discounted cash flow method. The first approach is highly arbitrary as, if market agents have non-homogeneous information sets or adopt different trading strategies, the benchmark used for comparison may be overvalued or undervalued. The second problem with this method is that industry or sector classifications become tricky as far as firms diversify their activities and develop new

is relatively less prone to measurement errors as it does not require the calculation of the book value of the firm in the evaluation of the fundamental. According to the DCF model - and under the assumption that the discounted cash flow to the firm is equal to net earnings[8] -, the "fundamental price-earning" ratio of the stock may in fact be written as:

$$MVE/X = \sum_{t=0}^{\infty} \frac{(1+E[g_t])^t}{(1+r_{CAPM})^t} \qquad (10)$$

where *MVE* is the firm equity value, *X* is the current cash flow to the firm,[9] $E[g_t]$ is the yearly expected rate of growth of earnings according to I/B/E/S consensus forecasts,[10] $r_{CAPM}= R_f + bE[R_m]$ is the discount rate adopted by equity investors, R_f represents the risk free rate, $E[R_m]$ the

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products or services which cannot be easily classified into traditional taxonomies (Kaplan and Roeback, 1995).
The problem with the second approach (residual income method), which is largely used in the literature (Lee-Myers-Swaminathan, 1999; Frankel-Lee, 1998), is that the formula for evaluating the fundamental value of a stock uses a balance sheet measure whose accuracy and capacity of incorporating changes in firm fundamental value is limited. A valuable example of this phenomenon may be given by inspecting descriptive evidence provided by Lee-Myers- Swaminathan (1999) who document the sharp uptrend in the price to book ratio which has risen three times between 1981 and 1996 for the Dow Jones Industrial Average. An interpretation for this result is that accounting methodologies lag behind in adjusting to changes in investors' market value assessments of firms whose share of intangible assets made by human and, more generally, immaterial capital is rising over time. Moreover, book values tend to be seriously affected by historical or market value acounting choices on nonrealised capital gains/losses. This means that, depending on the rule adopted, the book value is not independent from market over or underevaluation.
This is the reason why, following Kaplan-Roeback (1995), we prefer to use the DCF approach.

[8] The traditional DCF approach discounts dividends and not earnings. Many companies have recently started to postpone dividend payments at later stages of their life cycle (Campbell, 2000). In parallel, several authors use earnings rather than dividends to predict stock returns (Olhson, 1990 and 1995; Ang-Liu, 1998; Vuolteenaho, 1999; Fama-French, 1998; Lamont, 1998). Consider also that standard finance textbooks typically recommend DCF (Brealey-Myers, 2000; Ross, Westerfield and Jaffe, 2001) and that Graham-Harvey (2000) evaluating responses of 392 CFO's find that 74,9 percent use NPV techniques and that CFOs of larger firms apply DCF more than CFO of smaller firms.

[9] We are assuming in accordance with the literature, that, under perfect information and no transaction costs, the dividend policy does not affect the value of stocks (Miller-Modigliani, 1961).

[10] We use 1-year and 2-year ahead average earnings forecasts for the first two years and the long term average earning forecasts from the third to the sixth year.

expected stock market premium and b is the exposition to systematic nondiversifiable risk.

To calculate the fundamental value we consider the following "two stage growth" approximation of (10):

$$MVE = X + \sum_{t=1}^{5} \frac{X(1+E[g_U])^t}{(1+r_{CAPM})^t} + \frac{X(1+E[g_U])^6}{(r_{CAPM(TV)} - g_n)(1+r_{CAPM})^6} \qquad (11)$$

where MVE is the "two stage growth" equity market value, $E[g_U]$ is the expected yearly rate of growth of earnings according to the consensus of stock analysts. According to this formula the stock is assumed to exhibit excess growth in a first stage and to behave like the rest of the economy in a second stage. The second stage contribution to the MVE is calculated as a “terminal value” in the second addend of (11), where $r_{CAPM(TV)}$= $R_f + E[R_m]$ and g_n is the yearly rate of growth of earnings, after the sixth year, set equal to the expected perpetual nominal rate of growth of the economy.

The analytical definition of the DCF model imposes crucial choices on five key parameters: the risk free rate, the risk premium, the beta, the length of the first stage of growth and the nominal rate of growth of the terminal period.

For the risk free rate we use the yield on the three month US Treasury Bill.[11] For the risk premium we use the traditional measure of the historical difference in the rates of return of stocks and T-bonds for US

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[11] We choose a short term risk free rate to match its time length with the average time length of portfolio strategies which will be illustrated in sections 4 and 5. Results obtained when adopting a long term risk free rate (yield on the ten year Treasury Bill) are not substantially different from those presented in the paper and are omitted for reasons of space.

equity markets. Historical estimates of the risk premium for the period 1926-99 is 9.41% or 8.14% if we take respectively arithmetic averages or geometric averages of the stocks-T-bill spread. We choose an 8 percent risk premium which is also broadly consistent with estimates from Kaplan and Roeback (1995) indicating a value of 7.78 percent and Ibbotson and Associates (1996) reporting a similar value for the arithmetic average of historical risk premium. The third critical factor in the "two-stage" DCF formula is the terminal value of the stock. We fix at the sixth year the shift from the high growth period to the stable growth period. Sensitivity analysis on this threshold shows however that our choice is not crucial for the determination of the value of the stock.[12] This is because the positive impact on equity market value of an additional year of high growth is to be traded off with a heavier discount of the terminal value. In the terminal value it is assumed that the stock cannot grow more and cannot be riskier than the rest of the economy. The nominal average rate of growth of the economy in the terminal period, g_n, is prudentially set at 5 percent.[13] Finally, we alternatively try the estimation of a time varying beta[14] in a two year window of monthly observations and the choice of a unit beta.[15]

[12] Results are available from the authors upon request.
[13] In the sensitivity analysis which follows we test the robustness of our results to changes in the values of this parameter.
[14] There is a vast literature on sophisticated methods for estimating time varying beta. See for example Harvey-Siddique (2000) and Jagannathan-Wang (1996)
[15] The choice is reasonable given the size and representativeness of the S&P500 components and given several potential biases arising in beta estimates (noise, dependence from time varying leverage and business cycle conditions). The choice is nonetheless confronted with that of an estimated beta in our simulation (see sections 3-5).

The S&P500 aggregate fundamental to observed price ratios (also defined in the paper as the value price ratios) is built as an unweighted average of the value price ratios of each of the current S&P500 components.

3.2 Empirical findings

We test the theoretical model specified in section 2 for the S&P500 index. To do so we have a problem of selection bias since actual constituents do not correspond to the stocks which were part of the index at the beginning of the estimation period. If we want to replicate the index historically observed during the estimation period we need to replace current components with historical components. [16]

To estimate the trend p , we take in account the stock prices for each stock being part of the selected Index (S&P500). The Index monthly price is computed as an unweighted moving average of the 30-daily prices of Index constituents. We then define the S&P500 price trend as the filtered series of the Index price, obtained after removing low-frequency movements from the data.

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[16] The reconstruction of the S&P500 reveals that many of its current components were not present at the beginning of our estimation period. This reconstruction of the index reflects a significant change in the industry composition represented in it, with an increased weight of the high-tech with respect to traditional industries. Other newcomers are sector winners, affiliated to more traditional industries. The reconstruction is complex since only 309 constituents at the beginning of the sample period (February 1980) are still in the Index at the end of the period (December 2001). The list of current and historical components is available from the authors upon request.

To do so we construct a quasi-linear trend of price using a traditional Hodrick-Prescott (1980) filter, just to take in account financial cycles, removing low frequency movements ("non primary trends" according to chartists). As it is well known, the HP filter is a two-sided linear filter that computes the smoothed series of y_t by minimizing the variance of y_t around t_t, subject to a penalty that constrains the second difference of t_t. The HP filter chooses t_t to minimize: $\min_{t} \sum_{t=1}^{T} ((y_t - t_t)^2 + l((t_{t+1} - t_t) - (t_t - t_{t-1}))^2)$. The penalty parameter l controls the smoothness of the series and it depends on frequency data. We choose $l = 14{,}400$ for monthly data using a simple criterion of multiplying l with some power of the frequency adjustment, following Ravn and Uhlig (1997).

The gap p_t is calculated adopting the approach explained in section 3.1 and a fundamental value obtained with 8 percent risk premium and 3 percent nominal rate of growth in terminal value. To evaluate the sensitivity of our results to changes in parameters determining the fundamental value of the Index, we also provide an empirical test using an alternative fundamental with 6 percent risk premium and 5 percent nominal rate of growth in terminal value in Appendix A.

A simple descriptive inspection of the two series of the chartists' trend and of the deviation from the fundamental considered by fundamentalists in their trading activity, shows interesting results (Figures 1 and 2). The index appears to be significantly overvalued and the peak of the bubble at mid 2000 is clearly visible. Becchetti and Mattesini (2002) relate the dynamics of the deviation from the fundamental to other relevant economic events such as: i) the continuation of the effects of the 1979 oil shock (January -June 1980); ii) the crisis of the International Bank which manifests the weakness of the U.S. banking systems and preludes to the crisis of the S&L. This event is concurrent with an important change in the Fed's operating procedures (February 1982-August 1982); iii) the stock market crash of October 19, 1987, when the Dow Jones Index dropped by 22 percent in one day; iv) the effects of the international currency crisis leading to the devaluation of the Italian Lira and of the British pound (March 1992-October 1992); v) the Russian financial crisis (August 1998-October 1998).

It is also clear from the picture that an 8 percent risk premium implies an overall overevaluation of the fundamental during the sample period[17] and this is why we also estimate our model with a less severe hypothesis of a 6 percent risk premium.

What is impressive when we look at the dynamics of the chartists' trend (figure 2) is the strength of the chartists' momentum during the 1996-1999 period. Even more impressive is the fact that the trend itself behaves like a giant wave with its own and independent persistence irrespective of the deviation from the fundamental and of the variability of the latter in the same period. Our interpretation is therefore that the stock market boom is

[17] If we adopt the inverse approach of considering the fundamental equal to the observed stock value and solve the formula for the unknown implicit risk premium (or the risk premium which equals the fundamental to the observed value) we find that the implicit risk premium is slightly higher than 2 percent at the peak of the stock market boom.

mainly dominated by the chartists' pressure that overcomes fundamentalists' action. A relevant preliminary descriptive finding which seems to confirm the significance of our model is the difference in S&P500 mean monthly returns under a taxonomy of four different periods based on high and low values of, respectively, the trend and the deviation from fundamental variables (Tables 1 and 2).[18] The MMR under the high overevaluation (highgap83)/low trend period is about 0.3 percent, significantly lower than the MMR under the low overevaluation (lowgap83)/high trend period (1.4). Therefore the model correctly predicts that market returns are higher (lower) when our indicators suggest positive (negative) signals for fundamentalists and chartists. The difference is less significant than it could be in other periods as our figures show that in the time interval considered for our analysis there is hardly ever a situation in which the observed is below the fundamental value and the trend is positive and growing, but only situations in which the trend is positive and growing and the fundamental is only slightly overvalued (see Figures 1 and 2).

The model (7)-(9) is estimated by adopting a three stage least square approach.[19]

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[18] We consider as a threshold between low/high values of the two variables considered the median value. A sharper threshold (i.e. 30th/70th percentile) might have enhanced differences between subgroups but at the cost of eliminating too many observations in our sample.

[19] Three-stage least squares (3SLS) are the most appropriate technique when there is the risk that right-hand side variables are correlated with the error terms, and when both heteroskedasticity, and contemporaneous correlation in the residuals are likely to occur. The approach consists of applying TSLS (two-stage least squares) to the unweighted system, enforcing any cross-equation parameter restrictions. The estimates are used to form an estimate of the full cross-equation covariance matrix

Empirical findings on the S&P500 (Tables 3a-3b) show that all of the five parameters of the two equation model have the expected sign and are all significant.

This result appears robust to changes in the parameters adopted for the evaluation of the fundamental. It does hold for risk premia ranging from 6 to 8 percent, nominal rates of growth in the terminal period from 2 to 6 percent. Moreover, it is not sensitive to the replacement of estimated with unit betas.

Coefficients estimated with the discrete form (7)-(9), then, satisfy the stability conditions of the equilibrium of the differential equation system (5)-(8). In fact, for all values of m , since $bg'(0) > 1$, the local stability of equilibrium of the system (5)-(8) is possible because $m < m_0$ for all values of parameters calculated from the estimated coefficients (Sethi ,1996, proposition II). The trace of the *Jacobian* matrix, associated to the system (5)-(8) and evaluated at the equilibrium, is strictly negative for all values of m (see Appendix B).

This result ensures that our findings are compatible with stability conditions of the equilibrium of the continuous form of the model for all possible shares of wealth held by chartists, between zero and one.

4. Conclusions

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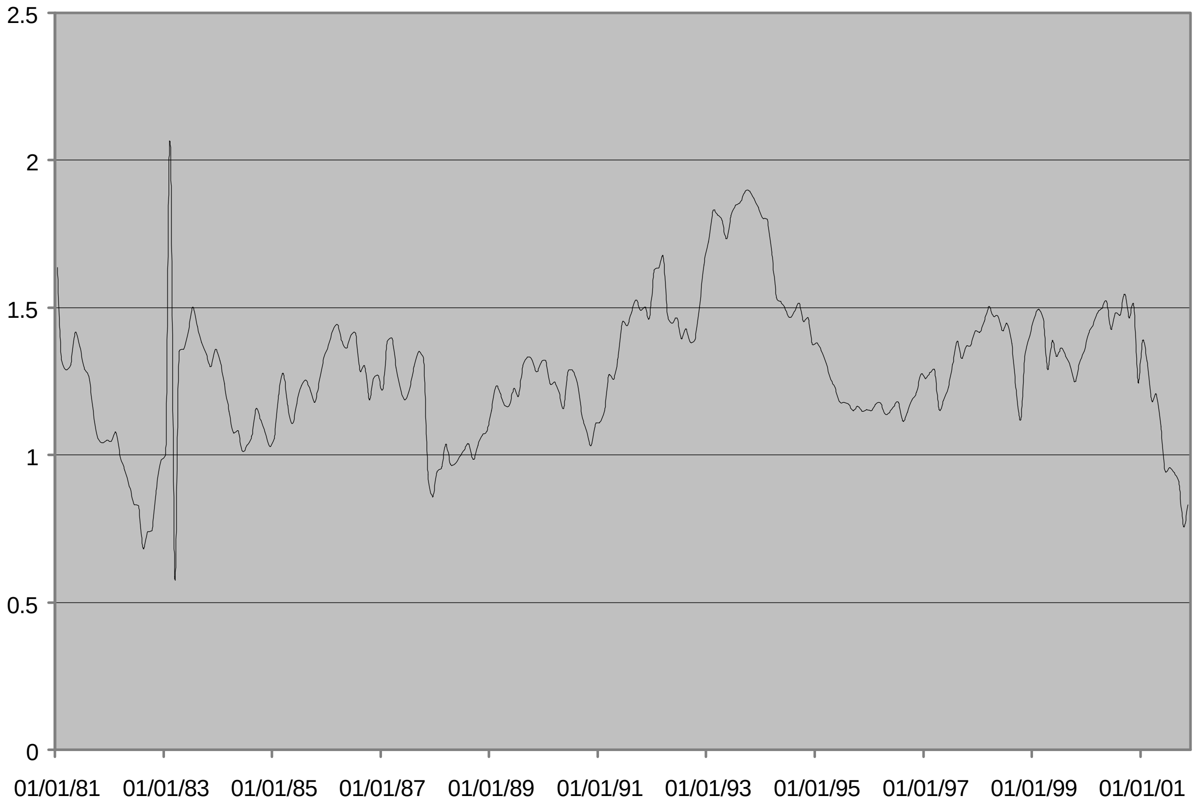
which, in turn, is used to transform the equations to eliminate the cross-equation correlation. TSLS is then applied to the transformed model.

Financial literature presents a vast number of theoretical stories on the effects of the interaction among traders with heterogeneous information sets, after the seminal contribution of Grossman (1981) demonstrating the impossibility of informationally efficient markets. Nonetheless, very few are the attempts of testing these models on financial data. Most of these models are too complex and do not end up with testable theoretical predictions. Some of them sacrifice more of the reality and therefore produce conclusions that can be verified by the empirical analysis. In this paper we choose one of them, the Sethi (1992) model, for empirical testing. The model examines the interplay of fundamentalists and chartists and derives from it a simple rule for the dynamics of stock returns and trends as affected by the trend itself and by deviations of observed price from fundamentals.

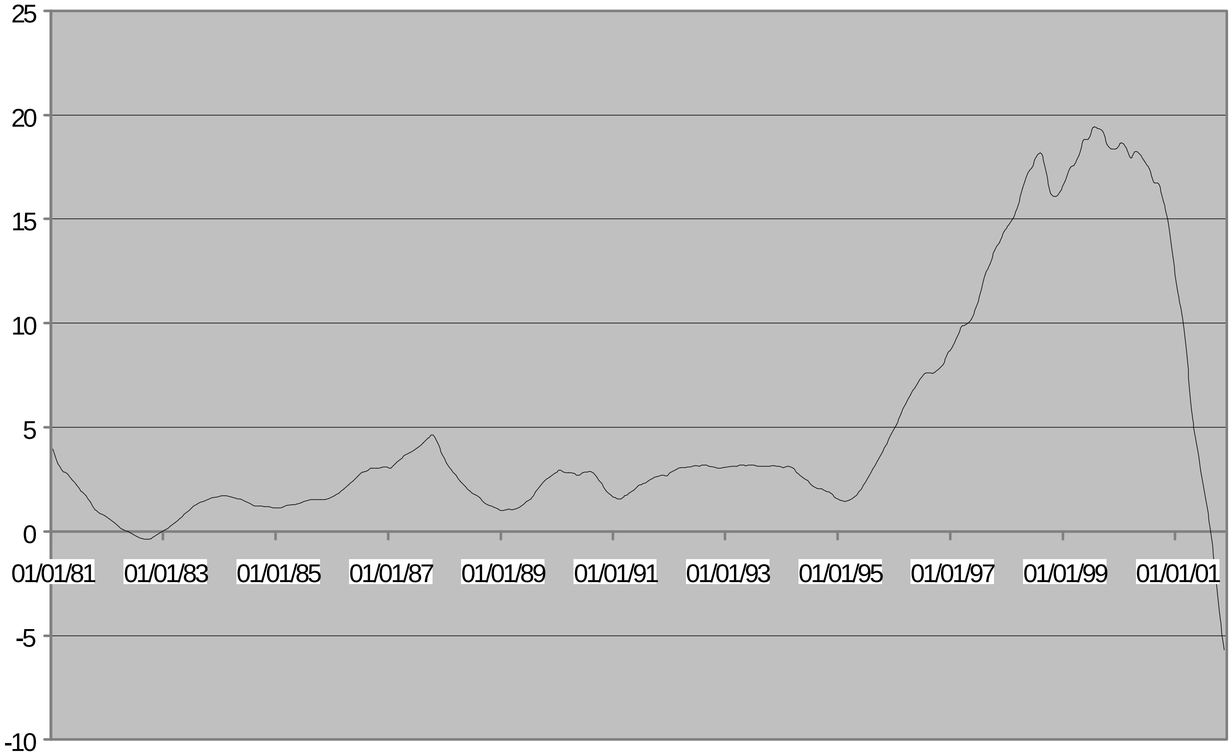
We propose in our paper a reasonable approximation for such fundamental using I/B/E/S Consensus forecasts and test the two equation system derived from the model by using average deviations from fundamentals calculated on the S&P500. Our results broadly support theoretical predictions showing that the dynamics of stock market price may be synthetically represented as the outcome of fundamentalists and chartists decisions, even in periods (such as that considered in our analysis) in which the pressure of one of the two groups is dominant.

Figure 1: Ratio between the observed and the fundamental stock price of the S&P500 index, p_t. *Fundamental value with 8 percent risk premium and 3 percent nominal rate of growth in terminal value, unit beta (when the index is equal to one the fundamental is equal to the observed value).*



Time period: jan 1981-dec 2001. Data source: Datastream.

Figure 2: Monthly trend in the stock prices, p_t (S&P500 index).



Variable legend: first difference of the HP trend of the monthly S&P500 price index. Time period: jan 1981-dec 2001. Data source: Datastream.

Table 1: Mean Monthly Returns (MMR) under four different scenarios based on relative chartists' and fundamentalists' pressure

Fundamental value with 8 percent risk premium and 3 percent nominal rate of growth in the terminal period (defined below as gap83).

	Highgap83(H p_t)	Lowgap83(L p_t)
Hightrend (Hp_t)	0.0131	**0.0148**
Lowtrend (Lp_t)	**0.0039**	0.0146

Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream.

Variables legend. Hp_t: periods in which the monthly trend in stock prices is *above* the median; H p_t: periods in which the difference between observed and fundamental price is *above* the median; Lp_t: periods in which the monthly trend in stock prices is *below* the median; L p_t: periods in which the difference between observed and fundamental price is *below* the median. The tables provides mean monthly in each of the following subperiods: (Hp_t, H p_t), (Hp_t, L p_t), (Lp_t, H p_t), (Lp_t, L p_t).

Table 2: T-test on mean difference (MMR Hp_t - L p_t > MMR Lp_t - H p_t).

Ho: mean(MR Hp_t - L p_t) = 0.0039	***Mean***	***Std. Err.***	***Std. Dev.***	***95% Conf. Interval***
MR Hp_t - L p_t	0.01481	0.0076	0.0453	-0.00075 - 0.03036
	t-stat.	***P-value***		
Ha: mean(MR Hp_t - L p_t) < 0.0039	1.4295	0.919		
Ha: mean(MR Hp_t - L p_t) ≠ 0.0039	1.4295	0.162		
Ha: mean(MR Hp_t - L p_t) > 0.0039	1.4295	0.081		
N. of obs.	35			
Degrees of freedom:	34			

We test whether mean monthly returns during time periods characterized by high trend and low gap in time subset (Hp_t, L p_t), are significantly different from those obtained during time periods characterized by low trend and high gap (Lp_t, H p_t). Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream. *Variables legend* :MR Hp_t - L p_t: monthly returns in the subperiod (Hp_t, L p_t).

Table 3a: Estimate of the (7)-(9) differential equation system
Fundamental value with 8 percent risk premium and 3 percent nominal rate of growth in the terminal period– Three stages least square (S&P500 index)

Dep. Var. Δp_t	***Coef.***	***z-stat***	***P-value***	***Dep. Var.*** Δp_t	***Coef.***	***z-stat***	***P-value***
$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$	0.0795	1.99	0.046	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$	0.0147	2.26	0.024
p_{t-1}	-0.0536	-1.97	0.048	p_{t-1}	-0.0394	-2.55	0.011
R^2	0.027			R^2	0.030		
c^2	3.9706			c^2	6.5127		
N. of obs.	144			N. of obs.	144		
Dep. Var. Δp_t	***Coef.***	***z-stat***	***P-value***	***Dep. Var.*** Δp_t	***Coef.***	***z-stat***	***P-value***
$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$	0.7609	3.04	0.002	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$	0.1805	3.80	0.000
p_{t-1}	-0.4227	-3.04	0.002	p_{t-1}	-0.1920	-2.65	0.008
p_{t-1}	-0.0084	-1.69	0.100	p_{t-1}	-0.0370	-3.57	0.000
R^2	0.088			R^2	0.122		
c^2	13.4336		0.003	c^2	18.8659		0.0003
N. of obs.	144			N. of obs.	144		
Endogenous variables				Δp_t, Δp_t			
Exogenous variables				$g(p_t)$, p_t, p_t			

Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream. *Variables legend* p_t: monthly trend in stock prices; p_t: difference between observed and fundamental price; Δp_t: one month change in the difference between observed and fundamental price; Δp_t: one month change in the stock price trend; $g(p_t) = \frac{\exp(p_t)}{1+\exp(p_t)} - \frac{1}{2}$, $g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$: chartists perceived trend in price movements under two different functional forms.

Table 3b: Model results

Fundamental value with 8 percent risk premium and 3 percent nominal rate of growth in the terminal period – Three stages least square (S&P500 index)– Least square (S&P500 index)

Dep. Var. Δp_t	***Coef.***	***t-stat***	***P-value***	***Dep. Var.*** Δp_t	***Coef.***	***t-stat***	***P-value***
$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$	0.7837	3.08	0.003	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$	0.1922	3.89	0.000
p_{t-1}	-0.4325	-2.67	0.008	p_{t-1}	-0.2049	-2.75	0.007
p_{t-1}	-0.0095	-1.74	0.084	p_{t-1}	-0.0398	-3.66	0.000
R^2	0.088			R^2	0.121		
$F(3, 141)$	4.54		0.004	$F(3, 141)$	6.47		0.000
N. of obs.	144			N. of obs.	144		

We estimate only the second equation of the system (7)-(9), solving it by substituting the (7) into (9). Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream.

Variables legend

p_t: monthly trend in stock prices; p_t: difference between observed and fundamental price; Δp_t: one month change of the difference between observed and fundamental price; Δp_t: one month change of the trend in stock prices; $g(p_t) = \frac{\exp(p_t)}{1+\exp(p_t)} - \frac{1}{2}$, $g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$: chartists perceived trend in price movements under two different functional forms.

APPENDIX A:

Table A1: : Estimate of the differential equation system (equations 7 and 9 in section 2))

Fundamental value with 6 percent risk premium and 5 percent nominal rate of growth in the terminal period – Three stages least square (S&P500 index)

Dep. Var. Δp_t	***Coeff.***	***z-stat***	***P-value***	***Dep. Var.*** Δp_t	***Coeff.***	***z-stat***	***P-value***
$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$	0.0478	1.89	0.059	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$	0.0038	1.33	0.184
p_{t-1}	-0.0473	-1.88	0.061	p_{t-1}	-0.015	-1.30	0.192
R^2	0.025			R^2	0.0136		
c^2	3.5836			c^2	1.8158		
N. of obs.	144			N. of obs.	144		
Dep. Var. Δp_t	***Coeff.***	***z-stat***	***P-value***	***Dep. Var.*** Δp_t	***Coeff.***	***z-stat***	***P-value***
$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$	1.0320	4.41	0.000	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$	0.1971	4.33	0.000
p_{t-1}	-0.8424	-4.01	0.000	p_{t-1}	-0.309	-3.18	0.001
p_{t-1}	-0.0152	-2.73	0.006	p_{t-1}	-0.041	-3.96	0.000
R^2	0.140		0.000	R^2	0.1368		
c^2	23.028		0.000	c^2	22.355		0.001
N. of obs.	144			N. of obs.	144		

Endogenous variables	Δp_t, Δp_t
Exogenous variables	$g(p_t)$, p_t, p_t

Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream. *Variables legend* p_t: monthly trend in stock prices; p_t: difference between observed and fundamental price; Δp_t: one month change of the difference between observed and fundamental price; Δp_t: one month change of the trend in stock prices; $g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$, $g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$: chartists perceived trend in price movements in two different functional forms.

Table A2: Model results

Fundamental value with 6 percent risk premium and 5 percent nominal rate of growth in the terminal period – Three stages least square (S&P500 index)

Dep. Var. Δp_t	***Coeff.***	***t-stat***	***P-value***	***Dep. Var.*** Δp_t	***Coeff.***	***t-stat***	***P-value***
$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} -$	1.0480	4.40	0.000	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$	0.2033	4.35	0.000
p_t	-0.8534	-4.00	0.000	p_t	-.3193	-3.21	0.002
p_t	-0.0158	-2.76	0.006	p_t	-0.0431	-3.98	0.000
R^2	0.14			R^2	0.14		
$F(3, 141)$	7.63		0.000	$F(3, 141)$	7.46		0.000
N. of obs.	144			N. of obs.	144		

We estimate only the second equation of the system (7)-(9), solving it by substitution. Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream.

Variables legend

p_t: monthly trend in stock prices; p_t: difference between observed and fundamental stock prices; Δp_t: one month change of the difference between observed and fundamental price; Δp_t: one month change of the trend in stock prices; $g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$, $g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$: chartists perceived trend in price movements in two different functional forms;

APPENDIX B. **Table B1: Stability conditions for the system of differential equation (5)-(8), using estimated coefficients**

gap (p_t)	m	$g(p_{t-1}) = 5 \cdot \tanh(p_{t-1}/5)$					$g(p_{t-1}) = \frac{\exp(p_{t-1})}{1+\exp(p_{t-1})} - \frac{1}{2}$				
		a	b	q	m_0	Jtr	a	b	q	m_0	Jtr
gap83	0.05	0.0370	97.618	0.0560	0.6063	-5.0479	0.0085	1799.9	0.0292	0.9327	-49.807
	0.1		48.809	0.1182	0.7665	-5.0479		899.94	0.0617	0.9670	-49.806
	0.2		24.404	0.2659	0.8829	-5.0477		449.97	0.1388	0.9851	-49.804
	0.3		16.27	0.4558	0.9296	-5.0481		299.98	0.2380	0.9913	-49.805
	0.4		12.202	0.7091	0.9545	-5.0478		224.99	0.3703	0.9944	-49.808
	0.5		9.7618	1.0636	0.9698	-5.0478		179.99	0.5554	0.9962	-49.807
	0.6		8.1348	1.5954	0.9801	-5.0478		149.99	0.8332	0.9975	-49.806
	0.7		6.9727	2.4818	0.9874	-5.048		128.56	1.2961	0.9984	-49.806
	0.8		6.1011	4.2545	0.9928	-5.0479		112.49	2.2218	0.9990	-49.804
	0.9		5.4232	9.5726	0.9968	-5.0479		99.994	4.9991	0.9996	-49.806
gap65	0.05	0.0415	95.004	0.0827	0.6694	-7.3066	0.0153	1346.5	0.0447	0.9215	-57.044
	0.1		47.502	0.1745	0.8120	-7.3068		673.24	0.0945	0.9614	-57.044
	0.2		23.751	0.3927	0.9084	-7.3066		336.62	0.2127	0.9825	-57.045
	0.3		15.834	0.6732	0.9456	-7.3066		224.41	0.3647	0.9898	-57.043
	0.4		11.876	1.0473	0.9651	-7.3071		168.31	0.5673	0.9935	-57.044
	0.5		9.500	1.5709	0.9770	-7.3065		134.65	0.8509	0.9957	-57.045
	0.6		7.917	2.3564	0.9849	-7.3067		112.21	1.276	0.9971	-57.048
	0.7		6.786	3.6655	0.9904	-7.3067		96.178	1.9854	0.9981	-57.043
	0.8		5.9378	6.2837	0.9945	-7.3067		84.155	3.4036	0.9989	-57.044
	0.9		5.278	14.138	0.9976	-7.3065		74.805	7.6581	0.9995	-57.044

Time period: jan 1989-feb 2001. Frequency data: monthly. Data source: Datastream.
Variables legend:Gap (p_t) : difference between observed and fundamental stock prices. *gap83* is the fundamental with 8 percent risk premium and 3 percent nominal rate of growth in the terminal period, while *gap65* is the fundamental with 6 percent risk premium and 5 percent nominal rate of growth in the terminal period; m : is the share of market wealth held by chartists; m_0 : is a bifurcation value of this wealth share; a , b , q are parameters of the system (5)-(8), constructed with the coefficients estimate with discretized form (7)-(9); *Jtr:* is the trace of the Jacobian matrix. evaluated at the equilibrium: $J = \begin{bmatrix} -b(1-m)q & bmg'(0) \\ -ab(1-m)q & a(bmg'(0)-1) \end{bmatrix}$; $g(p_t)$: chartists perceived trend in price movements in two different functional forms.

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