Demand Uncertainty, External Finance and the Business Cycles.*

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Abstract

The model presented in this paper addresses a simple,but critical, question: it is possible to generate business cycles **without productivity shocks**? To answer we use two-sector, two-good model, with agents heterogeneity. Capital is accumulated by each sector with resources generated in the corresponding sector (internal finance) and borrowing from the other one (external finance). There is one main result. A dynamic general equilibrium model where the only source of economic fluctuations comes from the demand side generates macroeconomic series consistent with the empirical evidences for the US economy.

Keywords: Demand Uncertainty, External Finance, Business Cycles, Keynesian Economics.

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1 Introduction

Business cycle research studies the causes and consequences of the recurrent expansions and contractions in aggregate economic activity occurring in most industrialized countries.

Kydland and Prescott (1982), and Long and Plosser (1983) first illustrated the promise of this approach, suggesting that one could build a succesfully business cycle model that involved market clearing, no monetary factors, and no rational for macroeconomic management. The 90's has been a decade of spectacular growth for this field of research. There has been produced so many theoretical and empirical contributions that it is not suggested to write down an exhaustive and complete bibliography.[1] We are quite comfortable in concluding that the dynamic stochastic general equilibrium model is firmly established as a laboratory in which modern macroeconomic theory is conducted.

But in the last few years there has been increasingly concern about the mechanism at the core of dynamic stochastic general equilibrium model: the idea the business cycles are mainly driven by large and cyclically volatile shocks to productivity, which are well represented by the Solow Residual as in Prescott (1986).

Estimation of Solow Residual presents many problems, as documented in many contributions (e.g. Burnside, Eichenbaum and Rebelo, 1996 or add references). Now recent studies have corrected the Solow Residuals for dismearusrements of inputs (e.g. unobserved effort and capacity utilization). These studies have caused productivity shocks to grow smaller and less cyclically volatile by introducing elements which moves sympathetically to economic cycle. Moreover, recent contributions show that positive technological shocks lead to decline in inputs (Gali (1999), Shea (1998), Basu, Kimball and Fernald (1999), and Nevill and Ramey (2002)). This can be seen as a problem within the business cycle literature, where productivity shocks represent the main source of fluctuations.[2]

Notice that this reasoning is not meant to bury the real business cycle research program, since a large number of developments and manipulation

[1] A valuable monitor for this ever-growing literature is provided by Christian Zimmerman web page (http://ideas.uquam.ca/qmrbc).

[2] For this reason, in order to match some key moment of the data RBC economist must allow for multiple sources of fluctuations, like stochastic taxation, stochastic government expenditure or debt.

there has been proposed to improve, often, successfully the performances of a real business cycle model (see Section 2 for a short review). It is meant, however, to incentives the research towards other, alternative sources of economic fluctuations.

The model presented in this paper addresses a simple question: it is possible to generate **business cycles without productivity shocks**? More precisely, we ask whether a model where (aggregate) uncertainty comes from the demand side is able to generate fluctuations consistent with the empirical evidences for the US economy. Quite surprisingly, the answer is yes.

The model is structured as two-sector, two-good economy, with agents heterogeneity. Capital is accumulated by each sector and with resources generated in the corresponding sector (internal finance) and borrowing from the other one (external finance). The economy is structured in the simplest possible way to focus the attention, as much as possible, to the role of shocks coming from the demand side, which is modelled using a state dependent utility function. Along the paper we refer to this mechanism as to *demand uncertainty*, *preference shocks*, or *taste shocks*.

There is one main result. A dynamic general equilibrium model where the only source of economic fluctuations comes from the demand side generates macroeconomic series consistent with the empirical evidences for the US economy.

Paper is organized as follows. Section 2 briefly surveys alternative sources of economic fluctuations, Section 3 presents the model, and Section 4 discusses selected issues about corner solutions and non-linear pricing. Section 5 discusses parameter calibration, and Section 6 analyzes numerical simulations. Finally Section 7 offers concluding comments.

2 Related Literature

There exist a huge literature concerning business cycles, and fluctuations in general, a survey of which is not a goal of this paper. It is, however, interesting to briefly discuss what sources of fluctuations have been used to generate aggregate fluctuations in the last twenty years.

To begin, notice that there exist an endless list of paper relying on *productivity shocks* as well as on *increasing returns and indeterminacy*. Fluctuations have been also generated introducing in the basic model *variable* *effort* or *variable capacity utilization* (e.g. Burnside et al., 1993, Bils and Cho, 1994, Basu and Shapiro, 1996), or *coordination failures - countercyclical mark-up* (e.g. Rotemberg and Saloner, Rotemberg and Woodford), or relying upon *income distribution* dynamics (e.g. Galor and Zeira, 1992, Danthine and Donaldson, 1992). Othe relevant sources of fluctuations are *monetary and fiscal policy* shock (e.g McGrattan et al. 1998, or Braun, 1994), or *nominal rigidities* and *imperfect competitions* (e.g. Hairault and Portier, 1993, Benassy, 1995, Gali, 1999).

Quite interestingly, the role of goods and services demand has not been exploited as well as other mechanisms. Indeed, there exist few contributions, as that of Stockman and Tesar (1994), of Bencivenga (1994), or Rotemberg and Saloner (1991). Stockman and Tesar work out an open economy model with taste, technology shocks and sector-specific capital, but without inter-country commerce. Bencivenga introduces a taste shifter between consumption and leisure, while Rotemberg and Saloner use countercyclical mark-up for solving labor market problems.

3 The Model

The model is structured as two-sector, two-good economy, with agents heterogeneity. To keep the model as simple as possible, we do not analyze here labor-leisure choice. Preference shocks are modelled using a state dependent utility function, which form changes across states and over time. Since there are no restriction to trade among agents, we solve a Benevolent Planner problem and, then, we characterize the equilibrium.

3.1 Consumption Set and Preferences.

In an economy populated by two large groups of individuals, representing shares $0 \leq \varphi \leq 1$ and $0 \leq (1-\varphi) \leq 1$ of total population, there exist two different goods (or commodity bundles), denoted by C_1 and C_2. First group consumers demand C_1 only, while those belonging to the second group demand just C_2. Assume that all individuals of each group are identical, while differing between groups.[3]

Moreover, there exist three states of nature, where the demand of a commodity, say C_2, fluctuates among a *high level* state ($\omega = h$), a *steady-state*

[3] An alternative interpretation without heterogeneity could consider φ as a weight assLciatated with C_1, and $1-\varphi$ as that associated with C_2 within the utility function.

level state ($\omega = m$), and a *low level* state ($\omega = l$). Dynamics is generated by a 3-state first order linear Markov Process (MP). Denote the time t, state ω value as $s(\omega)$, where $\omega = (l, m, s)$, and assume that $s(h) > s(m) > s(l)$. The associated transition matrix is defined as Π:

$$\Pi \equiv \begin{pmatrix} \pi_{\bullet\bullet} & \cdots \\ \vdots & \ddots \end{pmatrix} \tag{1}$$

where $\pi_{\bullet\bullet} \in (0, 1)$ denotes a transition probability. When $s = s(h)$, good demand is high, representing a boom phase for the economy. Instead when $s = s(m)$ and $s = s(l)$ defines a steady state and a recession cyclical moment, respectively. Finally, assume that the MP is not degenerate, in the sense that there are no absorbing states.[4]

Preferences are described by a state dependent felicity function:[5]

$$u(\mathbf{C}, s(\omega)) = \varphi u(C_1) + (1 - \varphi)u(C_2, s(\omega)) \tag{2}$$

where $\mathbf{C} = \langle C_1, C_2 \rangle$, $u(C_1) = \frac{C_1^{1-q_1}}{1-q_1}$, $u(C_2, s(\omega)) = \frac{s(\omega)C_2^{1-q_2}}{1-q_2}$ and s is a realization of the preference shock.[6] Parameters q_1 and q_2 denote the relative risk aversion (RRA) coefficients for each group.[7]

Assumption 1 *The instantaneous utility function (2) has extended expected utility (EEU) representation as from Savage (1972)* $E_t u(C_1, C_2; s) = \sum_{\kappa=1}^{3} u(C_1, C_2; s_\kappa)p_\kappa$, *where* $\{p_\kappa\}_{\kappa=1}^{3}$ *denote the conditional probabilities that system is in Regime* κ *at time t.*

[4] Alternatively, we can describe the same process with an AR(1) model, $s_{t+1} = \rho s_t + \epsilon_{t+1}$, where ϵ_{t+1} is a iid vector of normal random variables.

[5] To keep the problem simple we do not consider a labor-leisure choice in this formulation.

[6] Notice that s does not represent a change of the population composition (that is of the income distribution), but it is a preference shifter increasing (or reducing) the utility coming from C_2.

[7] Since we work with two different groups of consumers, a CES aggregator it is not used, as it should has been done, if we were working with one agent consuming two goods. Notice that $s(\omega)$ affects marginal utility of just one commodity. It can be showed that a model where preference shocks change the marginal utility of both commodities (i.e. $u = s(\omega)(\mathbf{C})$) is equivalent to one where sujective discount factor is stochastic.

3.2 Technologies.

Each good is produced with a sector-specific Cobb-Douglas technology, which differs in the capital share, $(\alpha_i)_{i=1}^2$ and in the average level of productivity, $(\lambda_i)_{i=1}^2 > 0$.

$$Y_1 = \lambda_1 K_1^{\alpha_1} \tag{3}$$
$$Y_2 = \lambda_2 K_1^{\alpha_2} \tag{4}$$

In this context firms choose production capabilities computing the expected value of consumers' demand, and capital stock is accumulated in each sector with resources generated in the corresponding sector (internal finance) and borrowing from the other one (external finance).

Capital accumulation constraints are modified to accomplish the existence of external finance flow, which time t value is denoted as T:

$$K_1' = (1 - \Omega_1)K_1 + \tilde{Z}_1 + T \tag{5}$$
$$K_2' = (1 - \Omega_2)K_2 + \tilde{Z}_2 - T \tag{6}$$

where the prime denotes a next period variable, $(\tilde{Z}_i)_{i=1}^2$ represent before-transfer investment flows, and T is defined as a net resource transfer from sector 2 to sector 1. Define the after transfer investment flows as $Z_1 = \tilde{Z}_1 + T$ and $Z_2 = \tilde{Z}_2 - T$.

When $T > 0$, model is transferring resources **from sector 2 to sector 1**; when $T < 0$ the direction of the flow is inverted, going from **sector 1 to sector 2**.[8] This formulation depicts a situation where capital can be accumulated with **internal** and/or with **external** finance. And since net transfer does not enter into the utility function, we are implicitly assuming that consumers are indifferent between accumulating capital either with internal finance, Z_i, or with external finance, T.

Finally, feasibility of the optimal program is ensured by the following two constraints:

$$C_1 + \tilde{Z}_1 \leq Y_1 \tag{7}$$
$$C_2 + \tilde{Z}_2 \leq Y_2 \tag{8}$$

[8] Notice that here we do not consider transfers of capital stocks, but of resources produced in sectors different from the final one. In fact King and Rebelo (1999) stresses that the low cyclical volatility of capital is often taken to imply that we can abstract from movement in capital in constructing a theory of economic fluctuations.

where C_1 and C_2 denotes final consumption flows. The intuition here is that each agent chooses how much to consume, and how much to invest of her own income. Next, it could be imagine that financial markets opens, where external finance contracts are signed to ensure that consumption profile. Resources are then transferred from one sector to the other, depending on the relative commodity demands.

3.3 Discussion and Timing of the Model

There are two possible interpretation for the two commodities of the model.

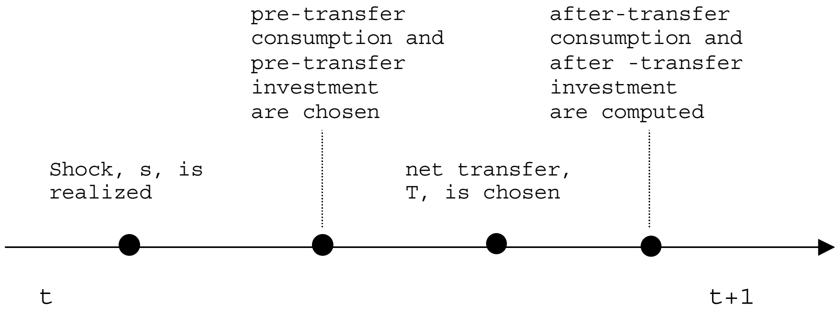
First, C_1 and C_2 could represent different qualities of the same commodity (e.g. the old and the new version of a certain good). If we consider the chipset industry, C_1 could represent a slower computer processing unit (CPU) than C_2 (for example C_1 can be a Pentium IV-1.6 GhZ, while C_2 a Pentium IV-1.8 GhZ); in the automobile industry, C_1 can represent a lower quality car model (e.g. a VW Golf IV Comfortline) than C_2 does (a VW Golf IV Highline). In this context, an increase of s would represent the introduction of a new model within the economy.

Alternatively, C_1 and C_2 could be interpreted as the basic and the luxury commodity, respectively. There exist evidences that basic commodity demands is more stable than that of luxury goods. Hence, because C_1 demand is not affected directly by preference shock $s(\omega)$, it is chosen to represent the basic good. Thus, C_2 represents luxury goods. The interpretation of the model is a critical issue, since it determines model's calibration (see Section 5). Next, timing of the model deserves some attention, too.

3.3.1 The Timing

The key issue to notice is that an external finance flow as modelled in equations (5) and (6) pools together the capital accumulation process, without adjustment costs, at least in this formulation. **Figure 1** presents the timing of the model.

Figure 1. Timing of the Model



Suppose that at time t the system is seating on its long run equilibrium. After a demand shock occurs, each consumer chooses a final consumption, $(C_i)_{i=1}^2$, and a before-transfer investment flow, $(\tilde{Z}_i)_{i=1}^2$. Then, they choose how much resource to transfer between sectors, that is T. In other words, consumer choose how much to consume, and then external finance flow is made to accomplish that choice. It is like having a market where contingent commodities (the net transfer, T) are traded to equate marginal utilities of consumption flows.[9] If, for example, one commodity is particularly desirable, the corresponding sector will increase its demand of external funds to accumulate capital, since its internal resources are allocated to consumption.

3.3.2 Mr. Keynes and the Neoclassics

This formulation where uncertainty comes from the demand side, gives a keynesian flavor to the model for two reasons.

First, notice firm choose how much to produce computing the expected value of the consumer demand, as well as in Keynes' General Theory. But

[9] Note that here we pool together the capital stocks, and that the transfer is made out od capital stock. This formulation differs from one where production is pooled together, where transfer would have been made out of output. Formally, the feasibility and the capital accumulation constraints would be changed as follow:

$$\begin{aligned} C_1 + Z_1 &\leq Y_1 + T \\ C_2 + Z_2 &\leq Y_2 - T \\ K_1' &= (1 - \Omega_1)K_1 + Z_1 \\ K_2' &= (1 - \Omega_2)K_2 + Z_2 \end{aligned}$$

there exist an difference, too. In the General Theory aggregate demand level might be (and usually it does) below its potential value. That happens because consumers hold money, without either consuming or investing, and thus reducing demand value. This does not happen, however, in our model where "un-used" resources are not held by consumers, but are transferred, as the external finance flow (T in the model's notation), to an other sector. This assumption implies that demand is always at the potential value, ensuring full employment within the economy.

Second, in General Theory business cycles originate from animal spirits perturbing the marginal efficiency of capital, while affecting, by this end, the marginal propensity to consumption (MPC). In our model, MPC is endogenous but preference shocks do affect it by changing the relative desirability between commodities.

3.4 Model Solution

Assume for moment that we have an interior solution. In this paper the analysis is restricted to cases where the model has an interior solution, even though it is not always the case. Under this assumption, which is guaranteed by proper restrictions, a standard Planner problem is solved, which value function, $\mathcal{W}(\mathbf{K}, s)$, after substituting the feasibility, (7) and (8), and the capital accumulation constraints, (5) and (6), satisfies

$$\begin{aligned} \mathcal{W}(\mathbf{K}, s) &= \sup_{\tilde{Z}_1, \tilde{Z}_2, T} \varphi \frac{\left(Y_1 - \tilde{Z}_1\right)^{1-q_1}}{1-q_1} + (1-\varphi) \frac{s\left(Y_2 - \tilde{Z}_2\right)^{1-q2}}{1-q_2} + \\ & \quad + \beta \sum_{j=1}^{3} \mathcal{W}'((1-\Omega_1)K_1 + \tilde{Z}_1 + T, (1-\Omega_2)K_2 + \tilde{Z}_2 - T, s')\pi_{ij} \end{aligned} \tag{9}$$

where $\pi_{ij} \in \Pi$ are transition probabilities, and $\mathbf{K} = \langle K_1, K_2 \rangle$ denotes the capital stock vector.

The optimal path for capital accumulation can be obtained by choosing sequences for consumption $(C_1(\mathbf{z}_t))_{t=1}^{\infty}$ and $(C_2(\mathbf{z}_t))_{t=1}^{\infty}$, investments $(\tilde{Z}_1(\mathbf{z}_t))_{t=1}^{\infty}$ and $(\tilde{Z}_2(\mathbf{z}_t))_{t=1}^{\infty}$, and transfer, $(T(\mathbf{z}_t))_{t=1}^{\infty}$, to maximize the value function (9), where $\mathbf{z}_t = \langle K_1, K_2, s \rangle$ denotes the state of the economy at time t. Manipulating the necessary and sufficient first order conditions (which are reported in Appendix A), we derive the following two Euler Equations (10) and (11), and a intra-temporal and intra-sector optimally condition, (12):

$$1 = \beta \sum_{j=1}^{3} \left[\left(\frac{C_1'}{C_1} \right)^{-q_1} R_1' \right] \pi_{ij} \quad (10)$$

$$1 = \beta \sum_{j=1}^{3} \left[\frac{s'}{s} \left(\frac{C_2'}{C_2} \right)^{-q_2} R_2' \right] \pi_{ij} \quad (11)$$

$$\varphi C_1^{-q_1} = (1-\varphi) s C_2^{-q_2} \quad (12)$$

where $R_1' = (\alpha_1 \lambda_1 (K_1')^{\alpha_1 - 1} + 1 - \Omega_1)$ and $R_2' = (\alpha_2 \lambda_2 (K_2')^{\alpha_2 - 1} + 1 - \Omega_2)$. Equations (10) and (11) are two traditional Lucas asset pricing equations.

3.4.1 The Non Stochastic Steady State

There is a unique non stochastic and stationary state that occurs in the economy when $s_t = s_m$ for all t. The first order condition can be used to describe this stationary state in a recursive manner. First, the capital accumulation efficiency implies that $r_i + \Omega_i = \alpha_i \lambda_i \bar{K}_i^{\alpha_i - 1}$, where $r_i = R_i - 1$, for $i = 1, 2$. Now, using the Euler Equations (10) and (11), we pin down the equilibrium value for capital stocks, $(\bar{K}_i)_{i=1}^2$

$$\bar{K}_i = \left(\frac{\alpha_i \lambda_i}{\frac{1}{\beta} - 1 + \Omega_i} \right)^{\frac{1}{1-\alpha_i}}$$

Next, using production functions, (3) and (4), we compute the equilibrium value for $(\bar{Y}_i)_{i=1}^2$

$$\bar{Y}_i = \lambda_i^{\frac{2-\alpha_i}{1-\alpha_i}} \left(\frac{\alpha_i}{\frac{1}{\beta} - 1 + \Omega_i} \right)^{\frac{\alpha_i}{1-\alpha_i}}$$

Substituting into the intra-sector optimally condition, (12), the feasibility constraints, (7) and (8), the state equations for capital stocks, (5) and (6), we find that the equilibrium value for $\bar{T}$ solves the following non linear equation:

$$\left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2 - \bar{T} \right) = \left(\frac{1-\varphi}{\varphi} s_m \right)^{\frac{1}{q_2}} \left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1 + \bar{T} \right)^{\frac{q_1}{q_2}} \quad (13)$$

Lemma 1 *When $q_1 = q_2 = q$, equation (13) has a closed form solution:*

$$\bar{T} = \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2\right) - \left(\frac{1-\varphi}{\varphi} s_m\right)^{\frac{1}{q}} \left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)}{1 + \left(\frac{1-\varphi}{\varphi} s_m\right)^{\frac{1}{q}}} \tag{14}$$

For all other cases, it has to be solved numerically. A matlab code is attached in the Appendix. Next, Proposition 1 shows that the net transfer is bounded above and below by the respective output.

Proposition 1 (Net Transfer's Bounds) *Net Transfers are bounded above by $\bar{T}_U = \left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2\right)$, and below by $\bar{T}_L = -\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)$.*

It is also interesting, and useful for some issues that are discussed below, to notice that the net transfer is monotone in the value of preference shock, s (Corollary 1).

Corollary 1 *Net Transfers monotonically increase in s, i.e. $\frac{\partial T}{\partial s} > 0$.*

Once we have a value for $\bar{T}$, we compute the *after-transfer* equilibrium value for investment flows, $(\bar{Z}_i)_{i=1}^2$, from (5) and (6):

$$\begin{aligned} \bar{Z}_1 &= \Omega_1 \bar{K}_1 - \bar{T} \qquad (15) \\ \bar{Z}_2 &= \Omega_2 \bar{K}_2 + \bar{T} \qquad (16) \end{aligned}$$

Finally equilibrium consumption $(\bar{C}_i)_{i=1}^2$, using the feasibility constraints, (7) and (8)

$$\begin{aligned} \bar{C}_1 &= \bar{Y}_1 - \Omega_1 \bar{K}_1 + \bar{T} \\ \bar{C}_2 &= \bar{Y}_2 - \Omega_2 \bar{K}_2 - \bar{T} \end{aligned}$$

3.4.2 The Competitive Equilibrium

When Theorem 2 holds, the model is fully characterized by the feasibility constraints, (7) and (8), the state equations for capital stocks, (5) and (6), the production functions (3) and (4), the Euler Equations, (10) and (11), and the intra-sector optimally condition, (12). Prices supporting the equilibrium, $\langle \mathbf{w}_t, \mathbf{r}_t \rangle$ are defined from production functions (3) and (4)

3.4.3 Transitional Dynamics

Transitional dynamics arises whenever the capital stock is different from its stationary state value, which happens, in this model, when consumer tastes change. Under Theorem 1, the model is stationary, and thus it is always optimal for the economy's capital stock to move monotonically towards the stationary level, for any initial level of capital (see Cass (1965) and Koopmans (1965)).

But, being highly non-linear, the system has no closed form solution. To study its stochastic properties we apply the well known procedure developed by King Plosser and Rebelo (1988a, b). In other words, we assume certainty equivalence, we linearize the system around its steady state, and we solve it applying linear approximations (e.g. Campbell 1994; Uhlig 1999). Appendix presents log-linearized model together with additional details.

4 Corner Solutions and Nonlinear Pricing

When solving the model in the previous section, it is assumed that it has an interior solution, which implies that capital is priced linearly, as we can see from the Euler Equations. This assumption is not neutral at all, since it has the important implication that consumption, investment and output are positive in equilibrium, and, even more remarkably, that their prices are equal in each sector, i.e. $p_{C_i} = p_{Z_i} = p_{Y_i}$ for $i = 1, 2$.

Quite interestingly, a closer look at the model shows that there exist several conditions under which it presents a corner solution (Proposition 2). Among the many possible scenarios, corner solutions arise when the relative size of one sector goes to zero ($\varphi \to 0$), or to one ($\varphi \to 1$), or when the relative desirability of C_2 collapses to zero ($s_l \to 0$), or becomes very large ($s_l \to \infty$). It happens too, if a sector becomes much more productive than the other ($\lambda_i \gg \lambda_j$ for $i, j = 1, 2$ and $i \neq j$).

Proposition 2 (Corner Solutions) *If either* $\varphi \to 1$, $\lambda_2 \to \frac{\Omega_1 K_1 + \Omega_2 K_2 - \lambda_1 K_1^{\alpha_1}}{K_2^{\alpha_2}}$ *or* $s \to 0$, *then* $\bar{C}_1 > 0$ *and* $\bar{C}_2 = 0$; *if, instead, either* $\varphi \to 0$, $\lambda_1 \to \frac{\Omega_1 K_1 + \Omega_2 K_2 - \lambda_2 K_2^{\alpha_2}}{K_1^{\alpha_1}}$ *or* $s \to \infty$, *then* $\bar{C}_2 > 0$ *and* $\bar{C}_1 = 0$.

Now, when the system reaches a corner solution, the equality among the price of consumption, of investment and of output does not hold anymore, since either consumption or investment of one sector is not demanded. It is clear that, in this case, the two quantities cannot have the same price. In this case, which is usually denoted as nonlinear pricing, equations (10), (11) and (12) do not fully characterize anymore the model equilibrium. This paper, however, does not focus on nonlinear pricing, while studying the business cycle implications of the model.

In this context, there exist at least two possible ways to ensure the existence of an interior solution. Either the parameter space can be restricted, by imposing conditions on each parameter and among parameters, or, the role of preference shock can be exploited to reach that goal. We follow the second alternative, which is illustrated below.

Roughly speaking, the underlining idea is the following. If changes in preferences are within certain *reflecting barriers* (see below for a definition), the model has an interior solution. The important issue to point out is that these extreme bounds, within which there are no corner solutions, are endogenous, while depending on the parameter space. In other words, each parameterization of the model implies a certain amplitude of the MP, given the transition matrix.

Definition 1 (Reflecting Barrier) *A reflecting barrier is defined as a state which, when crossed in a given direction, say downwards, holds a particle until a positive jump occurs and allows the particle to mover and resume the random walk. In this context two reflecting barriers are defined: a lower barrier,* s_{LOW}*, and an upper one,* s_{UP}*. The former is defined as the state,* $s = s_{LOW}$*, at which* $\bar{C}_2 = 0$ *and* $\bar{C}_1 > 0$*. The latter is symmetrically defined as the state,* $s = s_{UP}$*, at which* $\bar{C}_2 > 0$ *and* $\bar{C}_1 = 0$*.*

Theorem 2 proves in a constructive way that when the preference changes are restricted within some reflecting barriers (which values, s_{LOW} and s_{UP} are derived along the proof) the system has an interior solution.

Theorem 2 *If changes in preference are restricted within two endogenous reflecting barriers, denoted as* s_{LOW} *and* s_{UP}*, the system has an interior solution, in the sense that* $\bar{C}_1, \bar{C}_2 > 0$*. The lower and the upper barrier equals* $s_{LOW} = \frac{\varphi}{1-\varphi}\frac{\left(\lambda_2\bar{K}_2^{\alpha_2}-\Omega_1\bar{K}_1-\Omega_2\bar{K}_2\right)^{q_2}}{\left(\lambda_1\bar{K}_1^{\alpha_1}\right)^{q_1}}$*, and* $s_{UP} = \frac{\varphi}{1-\varphi}\frac{\left(\lambda_2\bar{K}_2^{\alpha_2}\right)^{q_2}}{\left(\lambda_1\bar{K}_1^{\alpha_1}-\Omega_1\bar{K}_1-\Omega_2\bar{K}_2\right)^{q_1}}$*, respectively.*

There are several issues concerning the barriers themselves, like symmetry, or the distance between them, which Corollary 2 analyzes.

Corollary 2 *As Theorem 1 proves, both the barrier are endogenous, depending on the technology levels* $(\lambda_i)_{i=1}^2$*, on the depreciation rates* $(\Omega_i)_{i=1}^2$*, on the relative shares of the two consumer groups* φ*, on the RRA coefficients* $(q_i)_{i=1}^2$*, and on the elasticities of capital into the production functions* $(\alpha_i)_{i=1}^2$.

For example, if the two sectors have a very large productivity gap, say $\lambda_1 \gg \lambda_2$, the distance between the bounds widens, or if the subjective discount rate, β, becomes smaller, the distance shrinks, too.

Finally, Corollary 3 derives a value for preference shock at which there are no transfers, denoted as $\hat{s}$.

Corollary 3 $\exists! s \equiv \hat{s} : \bar{T} = 0, \hat{s} \in \mathcal{S}$, *where* $\hat{s} = \frac{\varphi}{1-\varphi} \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2\right)^{q_2}}{\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)^{q_1}}$.

The intuition is as follow. Suppose that a certain parameterization implies WLOG, that $\bar{T} < 0$, meaning that sector 1 is transferring resources to sector 2. Now if the desirability of second good, s, becomes gradually smaller, then external finance reduces to zero, to become, eventually, positive. This results is backed by Corollary 1, which proves the monotonicity of net transfers in s. It is clear, thus, that the role of preference shock is quite important, while causing which sector is borrowing and which one is lending. More precisely, $\hat{s}$ divides the parameter space into two regions, denoted as $\mathcal{S}_l := (s_{LOW}, \hat{s})$ and $\mathcal{S}_u := (\hat{s}, s_{UP})$, where the equilibrium and the behavior of the model are quite different. If the actual value for s belongs in lower interval, $s \in \mathcal{S}_l$, then the net transfer will be positive, while in the other case, $s \in \mathcal{S}_u$, it turns out to be negative.

5 Parameter Calibration

The system of equations we use to compute the dynamic equilibria of the model depends on a set of 13 parameters, and on a Markov transition matrix. **Six** pertains to the supply side $(\alpha_i, \lambda_i, \Omega_i)_{i=1}^2$. **Four** belong to demand side (q_1, q_2, φ, β). The final **three** parameters represent the value of the stochastic process representing preference intensity for new quality goods in the three states, $(s(l),\ s(m),\ s(h))$. The model is calibrated for the US economy following King and Rebelo (1999a) and Stock and Watson (1998) over the sample 1953:1- 1996:4. We precisely detail our calibration procedure below.

1. **Supply side parameters**. We set α_1 and α_2 equal to .333, which is a standard value for the US capital share (see King and Rebelo, 1999a, and Stock and Watson, 1998). next, we set $\lambda_1 = 1$ and $\lambda_2 = 1.05$. There exist large evidences that new goods are produced using technologies that are more productive (see Somebody for a survey), in the sense that the same amount of capital produces more output. Finally, the rate of capital depreciation is chosen to be 10% per annum for both sectors (and thus $\Omega_1 = \Omega_2 = 0.025$), which is a standard value for the US economy.

2. **Demand side parameters**. To calibrate the relative size of the two groups, φ, notice that consumers investing in new quality goods can be comfortably imagined being *wealthy.* By the same token, people consuming old quality goods can be considered poorer, or using an analogous terminology, liquidity constrained. Ball (1986), Ham (1986), Zeldes (1987), Campbell and Mankiw (1989), Hubbard and Kashyap (1992), Rodrigues and Varvatsoulis (1994) estimate that the share of liquidity constrained consumers for the US economy lies between 75% and 54% of total population. In this model φ is calibrated equal to 0.75, and robustness of this choice is validate by carrying out a sensitivity analysis exercise.
   Dynan (1993), then, shows that liquidity constrained consumers have a coefficient of relative prudence (as defined in Kimball, 1990) lower that the unconstrained ones, while preferring a less variable consumption profile. This means that their coefficient of relative risk aversion should be larger than that of unconstrained agents. In this model the RRA coefficient of liquidity constrained consumers, q_1, is set equal to 2.1 and that of unconstrained agents, q_2, equals 1. Finally, β is set to 0.984, standard value for the US economy.

3. **Preferences shock parameterization**. $s(m)$ denotes the value of preference shock around which the system is linearized, and its calibration is critical for the model. Indeed, once $s(m)$ is chosen, the endogenous barriers within which the system has an interior solution, s_{LOW} and s_{UP} are determined. In this context $s(m)$ is interpreted as the increase in utility that derives from consuming a new quality commodity. Hence, for calibrating this parameter it is required to uncover, in some sense, consumer preferences for the new goods. To do this we use data from the personal computer (PC) industry.[10] As

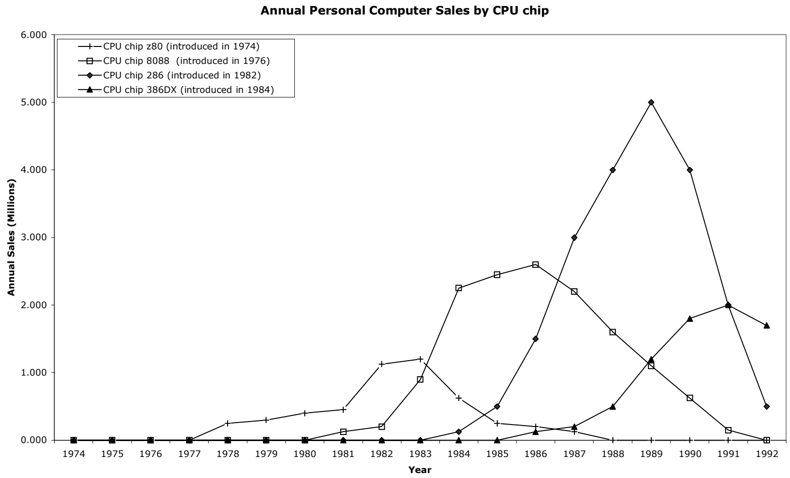
[10] A personal computer (PC) can be defined as a general-purpose, single-user machine

Bayus (1998) discusses, the most parsimonious way to describe the technology generations among personal computers is to compare their microprocessors or central processing units (CPUs). The CPU is the brain of the computer, because it contains the arithmetic and logic component as well as the core memory and control unit for the computer. Thus, CPU design determines the computer's overall power and performance. Using equation (12), we define C_2 and C_1 as the shipments (and thus consumption flows) of the new and the old CPUs. Notice that consumption of a new CPU is different from the introduction into the market of that CPU as new product. Indeed products shipments do not necessarily begin the same month or year when corresponding product is available on the market, as we can see from Figure 2. We can argue that shipment, and thus consumption, begins when consumers *discover* the availability of a new good (here notice the role of advertisement), and when they find it affordable. In this sense, a kind of revealed preferences are estimated, by looking at product shipments, and not at product proliferation.

Figure 2 depicts total annual sales for personal computers based on microprocessor chips (z80, 8088, 286, 386DX) over the sample (1974:1992). Note that this diagram (taken from Bayus, 1998) contains truncated data (e.g. 386DX personal computers continued to be sold after 1992).

Figure 2. Annual Personal Computer Sales

that is microprocessor based and can be programmed in a high-level language. Excellent historical reviews of the personal computer industry are given by Langlois (1992) and Steffens (1994).



Using data presented in Bayus (1998), and using equation (12), $s(m)$ is calibrated equal to 1.101. Reflecting barriers s_{LOW} and s_{UP} are then computed. This step is important since we want to bound preference shock fluctuations within these values. For this reason the upper value of the preference process, $s(h)$, is chosen equal to 90% of s_{UP}, and the lower one, $s(l)$, is set 10% bigger than s_{LOW}. In this case too, this choice is validate by a sensitivity analysis exercise.

4. Finally, the transition matrix of the Markov Process, Π, is calibrated as follows:

$$\Pi^* \equiv \begin{pmatrix} 0.50 & 0.50 & 0.00 \\ 0.25 & 0.50 & 0.25 \\ 0.00 & 0.50 & 0.50 \end{pmatrix}$$

Table 1 collects the calibrated parameters, denoted as starred.

Table 1: Calibrated Parameters

α_1^*	α_2^*	λ_1^*	λ_2^*	Ω_1^*	Ω_2^*
0.33	0.33	1.00	1.05	0.025	0.025
q_1^*	q_2^*	φ^*	β^*	$\hat{s}^{**}$	
2.10	1.00	0.75	0.984	1.101	

Table 1. Parameter notation is standard: α_i denotes K_i intensity, A_i is the average productivity of K_i, Ω_i represents capital i depreciation rate, for $i = 1, 2$. Next, q_i and φ represent the RRA coefficient and the relative size of consumers

consuming C_i, respectively, and β the subjective discount rate of the composite consumer.

Notice that the parameterization of the model is standard, as from King and Rebelo (1999), while differing in a critical aspect. In this model λ_i does not represent the equilibrium value for a productivity shock, but it is just a parameter, representing the average productivity of sector i. In this sense there are no productivity shocks in the economy.

6 Numerical Results

This section explores the long run, deterministic, equilibrium as well as model's dynamics.

6.1 The Long Run Deterministic Equilibrium

Table 2 presents long run (deterministic) equilibrium values for the benchmark model. Notice that the all variables are after-transfer, while representing the final consumption, the investment and the output flows.

Table 2: Equilibrium Individual Variables

$\bar{C}_1$	$\bar{C}_2$	$\bar{Z}_1$	$\bar{Z}_2$	$\bar{Y}_1$
2.36	2.23	0.35	0.68	2.70
$\bar{Y}_2$	$\bar{K}_1$	$\bar{K}_2$	$\bar{T}$	
2.91	19.83	21.33	0.15	

Note: $\bar{C}_1$ and $\bar{C}_2$ denote long run equilibrium consumption values, $\bar{Y}_1$ and $\bar{Y}_2$ equilibrium output flows, and $\bar{Z}_1$ and $\bar{Z}_2$ equilibrium after-transfer investment flows. Finally, $\bar{K}_1$ and $\bar{K}_2$ represent equilibrium capital stocks, and $\bar{T}$ equilibrium external finance flow from sector 2 to sector 1.

Since the new good sector is more productive, $(\lambda_1 > \lambda_2)$ its equilibrium capital stock is larger than that of the old sector, that is $\bar{K}_1 < \bar{K}_2$. Since $\bar{N}_1 = \bar{N}_2 = 1$, second sector output is sligthly larger than sector 1 one, $\bar{Y}_1 < \bar{Y}_2$.[11] In this context equilibrium net transfer, $\bar{T}$, is positive, meaning that sector 2 is transferring resources to sector 1 on a period by period

[11] This assumption is not strong at all, since the main intuitions of the model are preserved, as the section on Monotone Comparative Statics shows.

basis.[12] Notice, next, that equilibrium consumption of good 1 is larger than sector 2 counterparts, $\bar{C}_1 > \bar{C}_2$, while the inequality is inverted when looking at the investment flows, $\bar{Z}_1 < \bar{Z}_2$. That happens because sector 2, which demand is smaller than sector 1's, increases its before-transfer investment choice, a part of which, $\bar{T}$, is transferred to sector 1.[13]

It is now interesting to figure out what could be an intuition behind this result. Notice that the new quality goods ensure a better use of capital (since it technology is more productive), but since the corresponding demand is not as high as that for the old ones, innovator firms find optimal to invest some resources to the other sector. In this sense, the more advanced sector is financing the old one through external finance, while waiting for its own demand to pop up.

An other possible explanation departs from the new quality-old quality interpretation, while relying upon the concept of *certainty equivalent.* It can be imagined that consumers of group 2 prefer to consume less, but more surely. In this case, a net transfer would represent an additional device for smoothing consumption.

To compute, then, an equilibrium for the aggregate variables, we need to find the relative price of aggregate consumption, investment, production, capital, and net transfer. The following proposition derive the equilibrium relative price, $p_2 = \frac{(1-\varphi)sC_2^{-q_2}}{\varphi C_1^{-q_1}}$. And, since we are discussing an interior solution, it is easy to see that consumption, output, and investment of the same sector have equal price.[14]

Proposition 3 *Let $p = \langle p_1, p_2 \rangle$ be the price vector, where p_1 and p_2 denote price of the first and the second commodity. Normalizing $p_1 = 1$, the relative price supporting the competitive equilibrium is $\bar{p}_2 = \frac{(1-\varphi)sC_2^{-q_2}}{\varphi C_1^{-q_1}}$, provided we have an interior solution.*

The next step is to figure out which would be the price of the net transfer, $\bar{p}_T$, supporting a competitive equilibrium.

[12] Technically speaking, that happens because equilibrium value for preference shock around which the model is loglinearized, s_m^{**}, belongs in the interval $s_l^{**} < s_m^{**} < \hat{s}^{**}$.

[13] If we had chosen, however, $\hat{s}^{**} < s_m^{**} < s_l^{**}$, the picture should have been reversed, while $\bar{T}$ being negative in equilibrium.

[14] This would not be true when the system reaches a corner solution.

Corollary 4 *Suppose Theorem 2 holds (i.e. the system has an interior solution). Then, the price of net transfers, supporting the competitive equilibrium, is* $\bar{p}_T = \bar{p}_2 = \frac{(1-\varphi)sC_2^{-q_2}}{\varphi C_1^{-q_1}}$.

Using the results of previous two propositions, aggregate series are computed, and the corresponding equilibrium values are reported in **Table 3**.

Table 3. Long Run Equilibrium Aggregate Variables

	$\bar{C}$	$\bar{Z}$	$\bar{Y}$	$\bar{K}$	$\bar{N}$	$\bar{p}_2$
Standard RBC Model$^{(*)}$	2.21	0.5	2.70	19.83	1.00	-
(% of $\bar{Y}$)	81.7%	18.3%				
(% of $\bar{K}$)	11.1%	2.5%	13.6%			
This Model	4.32	0.95	5.26	38.60	1.00	0.88
(% of $\bar{Y}$)	82.1%	17.9%				
(% of $\bar{K}$)	11.2%	2.5%	13.6%			

Just notice that equilibrium relative price is lower than unity, $\bar{p}_2 = 0.88$, reflecting equilibrium values of disaggregated quantities. Aggregate equilibrium quantities are then expressed as a percentage of aggregate capital and aggregate output, in order to allow a easier comparison between the standard neoclassical model and ours. Notice that consumption and investment of our model equals that of the standard neoclassical model, as a percentage of aggregate capital stock. At contrary, our model allocates relatively more resources to consumption, out of total output, rather than to aggregate investment.

Notice that equilibrium consumption is higher, because the Planner (and thus the agents) has the opportunity of accumulating capital using either the internal or the internal finance channel. In other words, we could imagine that this mechanism enhances accumulation process, allowing to allocate a relatively *smaller amount* of resources to investment, but *use it better*.

6.2 Stochastic Simulations

To simulate the model, first a restricted Markov process for preference shock has to be generated. This process is restricted in the sense that it must fluctuate within the barriers computed in Section 4, and denoted as s_{LOW} and s_{UP}. The model is then simulated, and the generated series are analyzed, while focusing on volatility measures (**Table 4** and **5**) and on contemporaneous correlations among all the variables (**Table 6** and **7**).

6.2.1 Individual Series

Table 4: Volatility Individual Series

$\sigma^*_{Y_1}$	$\sigma^*_{Y_2}$	$\sigma^*_{C_1}$	$\sigma^*_{C_2}$	$\sigma^*_{Z_1}$	$\sigma^*_{Z_2}$
0.68	0.65	0.30	0.58	2.65	1.92
σ^*_{T}	$\sigma^*_{K_1}$	$\sigma^*_{K_2}$	$\sigma^*_{R_1}$	$\sigma^*_{R_2}$	
135.46	2.04	1.94	0.0651	0.0856	

Note. $\sigma^*_{Y_i}$, $\sigma^*_{C_i}$, $\sigma^*_{Z_1}$, $\sigma^*_{K_i}$ and $\sigma^*_{R_i}$ denote the standard deviation of output i, consumption i, investment i, capital i, and of return to capital i, respectively. Finally σ^*_T is the standard deviation of net transfers.

According to Table 4, the model generates substantial propagation of the shock, while presenting a relatively higher volatility of investment flows of sector 1, $\sigma^*_{Z_1}$, a lower volatility for $\sigma^*_{Z_2}$, and a very high volatility of external financial flows. Comparing then consumption and income flows volatilities, we have that $\sigma^*_{Y_1} > \sigma^*_{C_1}$, and that $\sigma^*_{Y_2} > \sigma^*_{C_2}$, which results are both consistent with consumption smoothing. Notice next $\sigma^*_{C_1} < \sigma^*_{C_2}$ consistently with the higher risk aversion of liquidity constrained consumers ($q_1 > q_2$). Finally, it is interesting to underline that the returns to capital are quite volatile, which is a important improvement upon the standard RBC model.

Table 5, then, presents the correlation among the individual series generated with the model, denoted as $\rho\,(x_i, x_j)$.

Table 5: Correlations Individual Series

	C_1	C_2	Z_1	Z_2	Y_1	Y_2	T	K_1	K_2
C_1	1.0000	**-0.9821**	0.2679	-0.7176	0.8524	-0.8259	0.2783	0.9995	-0.9924
C_2		1.0000	-0.3581	0.7460	-0.8824	0.8703	-0.1809	-0.9917	0.9993
Z_1			1.0000	**-0.7548**	0.7044	-0.7291	-0.8535	0.2760	-0.3007
Z_2				1.0000	-0.9663	0.9772	0.4699	-0.7036	0.7351
Y_1					1.0000	**-0.9957**	-0.2350	0.8600	-0.8775
Y_2						1.0000	0.2725	-0.8363	0.8620
T							1.0000	0.2938	-0.2528
K_1								1.0000	-0.9957
K_2									1.0000

Note. See Table 2.

The two sectors behave countercyclically, being the correlation between the consumption flows, $\rho(C_1, C_2)$, the investment flows, $\rho(Z_1, Z_2)$, and the individual productions, $\rho(Y_1, Y_2)$, negative. It is interesting to notice, moreover, that $\rho(C_1, Z_1)$ is much lower , while $\rho(C_2, Z_2)$ is large and positive. In other words, when the demand for C_2 goes up, it is convenient to accumulate capital in the corresponding sector, while relying on the **external finance channel**. Since relative desirability of consumption goods moves in favor of C_2 , it is convenient to use output produced in sector 2, Y_2, for consumption mainly, while accumulating K_2 using resources produced in sector 1, which demand is relatively lower. Net transfers are positively correlated with C_1, with Z_2, and with Y_2, while presenting a negative correlation with the other variables. The correlation of net transfer with remaining variables confirms this claim.

Example 3 *To understand more clearly the behavior of the economy, suppose that the demand for C_2 increases. According to the correlation matrix, C_1 falls, since the relative desirability between the two goods changes ($\rho(C_1, C_2) = -0.98$). At the same time, net transfers gets lower, and eventually become negative, financing in this way sector 2 capital accumulation with resources originating in sector 1. Hence K_1 is reduced, and K_2 increases ($\rho(K_1, K_2) = -0.98$). Notice that in this context Z_1 increases, while Z_2 decreases, since $\rho(Z_1, Z_2) = -0.75$. From this picture we observe a cross-sector financing of capital accumulation, that is what we can interpret as the* ***external finance accumulation channel****.*

6.2.2 Aggregate Series

We, then, move to analyze the behavior of the aggregate series. The first row of **Table 6** presents the standard deviations of the actual data, the second the statistics generated with a standard (one sector) RBC model. The third and the fourth rows report the volatility of the series generated by our model: precisely, the third presents the volatility measures of the raw series, while the fourth focuses on the Hodrick-Prescott (HP) filtered ones.

Table 6: Volatility Aggregate Series

	σ_C	σ_Z	σ_Y
Actual Data$^{(*)}$	1.45	5.30	1.81
Standard RBC Model$^{(*)}$	0.61	4.09	1.39
This model (Raw Series)	0.72	3.00	0.84
This model (HP-Filtered Series)	1.62	5.54	2.04

Table 5. (*) See King and Rebelo (1999), p. 9.

A casual comparison with the actual data, and the moments generated by the standard RBC model makes clear that a very simple model where uncertainty comes from the demand (preference) side generates series consistent with the actual data.

Finally, **Table 7** presents the correlation among aggregate, raw and HP-filtered, series and output.

Table 7 Contemporaneous Correlations with Output

	C	Z	Y
Actual Data	0.88	0.80	1.00
HP-filtered data	0.92	0.91	1.00
Raw data	0.90	0.71	1.00

Again the results are consistent with stylized facts of macroeconomic time series for the US economy. It is interesting to underline that aggregate consumption and investment are positively correlated, $\rho(C, Z) = 0.26$. This outlines that in this model the income effects dominates over the substitution effect, which is, however, the dominant one in models where fluctuations originate from the supply side. The other correlations are consistent with our intuition.

7 Conclusions

This paper originates from many unsatisfactory evidences about productivity shocks and real business cycle models. A two-sector, two-good economy, with agents heterogeneity, where business cycles are generated without productivity shocks, is presented in this paper.

There is one main result. A dynamic general equilibrium model where the only source of economic fluctuations comes from the demand side generates macroeconomic series consistent with the empirical evidences for the US economy.

Notice, to conclude, that there exist many developments originating from this models, like the analysis of non-linear pricing, the introduction of labor-leisure choice, or generalizations like introduction of external finance into consumer preferences.

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8 Appendix

8.1 A. Derivation of First Order Conditions

$$
\begin{aligned}
\mathcal{W}(\mathbf{K}, s) \; &= \; \sup_{\tilde{Z}_1, \tilde{Z}_2, T} \varphi \frac{\left(Y_1 - \tilde{Z}_1\right)^{1-q_1}}{1-q_1} + (1-\varphi) \frac{s\left(Y_2 - \tilde{Z}_2\right)^{1-q2}}{1-q_2} + \qquad (17) \\
&+\beta \sum_{j=1}^{3} \mathcal{W}'((1-\Omega_1)K_1 + \tilde{Z}_1 + T, (1-\Omega_2)K_2 + \tilde{Z}_2 - T, s')\pi_{ij}
\end{aligned}
$$

where $\pi_{ij} \in \Pi$ are transition probabilities, and $\mathbf{K} = \langle K_1, K_2 \rangle$ denotes the capital stock vector.

The n/s focs are

$$
\begin{aligned}
X_1 \; &: \; 0 = -\varphi C_1^{-q_1} + \beta \int W_1'(\mathbf{K}, s') dF(s', s) \\
X_2 \; &: \; 0 = -(1-\varphi) s C_2^{-q_2} + \beta \int W_2'(\mathbf{K}, s') dF(s', s) \\
T \; &: \; 0 = -\varphi C_1^{-q_1} + (1-\varphi) s C_2^{-q_2} + \beta \int W_1'(\mathbf{K}, s') dF(s', s) - \beta \int W_2'(\mathbf{K}, s') dF(s', s) \\
K_1 \; &: \; W_1(\mathbf{K}, s) = \varphi C_1^{-q_1} F_1(K_1) + (1-\Omega_1)\beta \int W_1'(\mathbf{K}, s') dF(s', s) \\
K_2 \; &: \; W_2(\mathbf{K}, s) = (1-\varphi) s C_2^{-q_2} F_1(K_2) + (1-\Omega_2)\beta \int W_2'(\mathbf{K}, s') dF(s', s)
\end{aligned}
$$

We can rewrite $W_1(\mathbf{K}, s)$ and $W_2(\mathbf{K}, s)$ as $W_1(\mathbf{K}, s) = \varphi C_1^{-q_1} \left[F_1(K_1) + (1-\Omega_1)\right]$ and $W_2(\mathbf{K}, s) = (1-\varphi) s C_2^{-q_2} \left[F_1(K_2) + (1-\Omega_2)\right]$.

Manipulating then the foc(T), it can be written either as

$$\varphi C_1^{-q_1} = (1-\varphi) s C_2^{-q_2}$$

Summarizing, equilibrium is characterized by the following focs

$$
\begin{aligned}
1 \; &= \; \beta \int \left[\left(\frac{C_1'}{C_1} \right)^{-q_1} R_1' \right] dF(s', s) \\
1 \; &= \; \beta \int \left[\frac{s'}{s} \left(\frac{C_2'}{C_2} \right)^{-q_2} R_2' \right] dF(s', s) \\
\varphi C_1^{-q_1} \; &= \; (1-\varphi) s C_2^{-q_2}
\end{aligned}
$$

8.1.1 The complete model

The system is characterized as follows:

Number of Equations: 12

Number of Variables: 12

State Variables (**3**): K_1, K_2, s.

Control Variables (**9**): $C_1, C_2, Z_1, Z_2, T, Y_1, Y_2, R_1, R_2$.

Set of Deterministic Equation

1 $C_1 + X_1 + T \leq Y_1$

2 $C_2 + X_2 - T \leq Y_2$

3 $Y_1 = A_1 K_1^{\alpha_1}$

4 $Y_2 = A_2 K_1^{\alpha_2}$

5 $K_1' = (1 - \Omega_1) K_1 + X_1 + T$

6 $K_2' = (1 - \Omega_2) K_2 + X_2 - T$

7 $\varphi C_1^{-q_1} = (1 - \varphi) s C_2^{-q_2}$

8 $R_1 = \alpha_1 A_1 K_1^{\alpha_1 - 1} + 1 - \Omega_1$

9 $R_2 = \alpha_2 A_2 K_2^{\alpha_2 - 1} + 1 - \Omega_2$

Set of Expectation Equations

10 $1 = \beta \int \left[\frac{s'}{s} \left(\frac{C_2'}{C_2} \right)^{-q_2} R_2' \right] dF(s', s)$

11 $1 = \beta \int \left[\left(\frac{C_1'}{C_1} \right)^{-q_1} R_1' \right] dF(s', s)$

Exogenous Stochastic Process

12 s' is a Linear Markov Process

8.1.2 The Log-linear Model

1. $0 = \left[\bar{Y}_1\right] \hat{y}_1 + \left[-\bar{C}_1\right] \hat{c}_1 + \left[-\bar{Z}_1\right] \hat{z}_1$

2. $0 = \left[\bar{Y}_2\right] \hat{y}_2 + \left[-\bar{C}_2\right] \hat{c}_2 + \left[-\bar{Z}_2\right] \hat{z}_2$

3. $0 = \left[-\bar{Y}_1\right] \hat{y}_1 + \left[\alpha_1 A_1 \bar{K}_1^{\alpha_1}\right] \hat{k}_1$

4. $0 = \left[-\bar{Y}_2\right] \hat{y}_2 + \left[\alpha_2 A_2 \bar{K}_2^{\alpha_2}\right] \hat{k}_2$

5. $0 = \left[-\bar{K}_1\right] \hat{k}_1' + \left[(1 - \Omega_1)\bar{K}_1\right] \hat{k}_1 + \left[\bar{Z}_1\right] \hat{z}_1 + \left[\bar{T}\right] \hat{t}$

6. $0 = \left[-\bar{K}_2\right] \hat{k}_2' + \left[(1 - \Omega_2)\bar{K}_2\right] \hat{k}_2 + \left[\bar{Z}_2\right] \hat{z}_2 - \left[\bar{T}\right] \hat{t}$

7. $0 = \left[\bar{s}(1 - \varphi)\bar{C}_2^{-q_2}\right] \hat{s} + \left[-(1 - \varphi)\bar{s}\bar{C}_2^{-q_2} q_2\right] \hat{c}_2 + \left[\varphi \bar{C}_1^{-q_1} q_1\right] \hat{c}_1$

8. $0 = [-1]\, \hat{r}_1 + \left[\beta \alpha_1 A_1 \bar{K}_1^{\alpha_1 - 1}(\alpha_1 - 1)\right] \hat{k}_1$

9. $0 = [-1]\,\hat{r}_2 + \left[\beta\alpha_2 A_2 \bar{K}_2^{\alpha_2-1}(\alpha_2-1)\right]\hat{k}_2$

10. $0 = E_t\left([-q_2]\,\hat{c}_2' + [q_2]\,\hat{c}_2 + [+1]\,\hat{s}' + [-1]\,\hat{s} + [+1]\,\hat{r}_2'\right)$

11. $0 = E_t\left([-q_1]\,\hat{c}_1' + [q_1]\,\hat{c}_1 + [+1]\,\hat{r}_1'\right)$

12. s' is generated using transition matrix Π

8.2 C. Proofs and Derivations

Proof of Lemma 1. Easy. Just set $q_1 = q_2 = q$ into 13, then solve for $\bar{T}$. ■

Proof of Corollary 1. Consider (13), and differentiate it with respect to s_m:

$$\frac{\partial \bar{T}}{\partial s_m} = -\frac{1-\varphi}{q\varphi}\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1-q}{q}}\left(\lambda_1\bar{K}_1^{\alpha_1} - \Omega_1\bar{K}_1\right)\frac{1+2\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1}{q}}+\left(\lambda_2\bar{K}_2^{\alpha_2}-\Omega_2\bar{K}_2\right)}{\left(1+\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1}{q}}\right)^2}.$$

Now because $\frac{1+2\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1}{q}}+\left(\lambda_2\bar{K}_2^{\alpha_2}-\Omega_2\bar{K}_2\right)}{\left(1+\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1}{q}}\right)^2} > 0$ and $\frac{1-\varphi}{q\varphi}\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1-q}{q}}\left(\lambda_1\bar{K}_1^{\alpha_1} - \Omega_1\bar{K}_1\right) >$ 0 too, there are no economically meaningful values for $\mathbf{K} = \langle K_1, K_2\rangle$ such that $\frac{\partial \bar{T}}{\partial s_m} = 0$. ■

Proof of Proposition 1. Let $q_1 = q_2$. Then Lemma 1 holds, and $\bar{T} = \frac{\left(\lambda_2\bar{K}_2^{\alpha_2}-\Omega_2\bar{K}_2\right)-\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1}{q}}\left(\lambda_1\bar{K}_1^{\alpha_1}-\Omega_1\bar{K}_1\right)}{1+\left(\frac{1-\varphi}{\varphi}s_m\right)^{\frac{1}{q}}}$. Consider the following cases: $\varphi \to 1$, $\varphi \to 0$, $s_m \to 0$, $s_m \to \infty$. Substituting $\varphi = 1$ into $\bar{T}$ we have $\lim_{\varphi=1}\bar{T} = \lambda_2\bar{K}_2^{\alpha_2} - \Omega_2\bar{K}_2$. When $\varphi \to 0$, then $\lim_{\varphi=1}\bar{T}$ is an indeterminate form of the kind $\frac{\infty}{\infty}$. An application of the L'Hôpital rule leads to $\bar{T} = \left(\lambda_1\bar{K}_1^{\alpha_1} - \Omega_1\bar{K}_1\right)$. When $s_m \to 0$, we get, by direct substitution, $\lim_{\varphi=1}\bar{T} = \lambda_2\bar{K}_2^{\alpha_2} - \Omega_2\bar{K}_2$, while when $s_m \to \infty$ L'Hôpital rule is need again to compute the limit $\lim_{\varphi=1}\bar{T} = -\left(\lambda_1\bar{K}_1^{\alpha_1} - \Omega_1\bar{K}_1\right)$. ■

Proof of Proposition 2. The proposition is proved by verifying that substituting into the system of equations the conditions stated above, the model presents a corner solution. If $\varphi \to 1$ or $s \to 0$, then $\bar{T} = \lambda_2\bar{K}_2^{\alpha_2} - \Omega_2\bar{K}_2$ (Proposition 1), which, substituted into (15) and (16) implies that after-transfer investment flows are $\bar{Z}_1 = \Omega_1 K_1 + \Omega_2 K_2 - \lambda_2 K_2^{\alpha_2}$ and $\bar{Z}_2 = \lambda_2 K_2^{\alpha_2}$

and that consumption flow are $\bar{C}_1 = \lambda_2 K_2^{\alpha_2} + \lambda_1 K_1^{\alpha_1} - \Omega_1 K_1 - \Omega_2 K_2$ and $\bar{C}_2 = 0$. By construction, when $\lambda_2 = \frac{\Omega_1 K_1 + \Omega_2 K_2 - \lambda_1 K_1^{\alpha_1}}{K_2^{\alpha_2}}$ we have that $\bar{T} = \lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2$, which implies that $\bar{C}_1 = \lambda_2 K_2^{\alpha_2} + \lambda_1 K_1^{\alpha_1} - \Omega_1 K_1 - \Omega_2 K_2$ and $\bar{C}_2 = 0$. Using the same technique, Proposition 1 shows that if $\varphi \to 0$, or $s \to \infty$ then $\bar{T} = -\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)$. Substituting again this value into (15) and (16), we after-transfer investments equal $\bar{Z}_1 = \lambda_1 K_1^{\alpha_1}$ and $\bar{Z}_2 = \Omega_1 K_1 + \Omega_2 K_2 - \lambda_2 K_2^{\alpha_2}$, and after-transfer consumption $\bar{C}_2 = \lambda_2 K_2^{\alpha_2} + \lambda_1 K_1^{\alpha_1} - \Omega_1 K_1 - \Omega_2 K_2$ and $\bar{C}_1 = 0$. Finally, when $\lambda_1 = \frac{\Omega_1 K_1 + \Omega_2 K_2 - \lambda_2 K_2^{\alpha_2}}{K_1^{\alpha_1}}$, by construction we have that $\bar{T} = -\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)$, which implies, again, that $\bar{C}_2 = \lambda_2 K_2^{\alpha_2} + \lambda_1 K_1^{\alpha_1} - \Omega_1 K_1 - \Omega_2 K_2$ and $\bar{C}_1 = 0$. ■

Proof of Theorem 1. The theorem is proved in a constructive way. First, we prove that when s reaches a lower barrier, denoted as s_{LOW} then $\bar{C}_2 = 0$ and $\bar{C}_1 > 0$ (Lemma 1), and that when s reaches an upper barrier, denoted as s_{UP} then $\bar{C}_2 > 0$ and $\bar{C}_1 = 0$ (Lemma 2).

Claim 1 $s = \frac{\varphi}{1-\varphi} \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_1 \bar{K}_1 - \Omega_2 \bar{K}_2\right)^{q_2}}{\left(\lambda_1 \bar{K}_1^{\alpha_1}\right)^{q_1}} \Rightarrow \bar{C}_2 = 0$ *and* $\bar{C}_1 = \lambda_2 K_2^{\alpha_2} + \lambda_1 K_1^{\alpha_1} - \Omega_1 K_1 - \Omega_2 K_2$. *Substituting* $s = \frac{\varphi}{1-\varphi} \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_1 \bar{K}_1 - \Omega_2 \bar{K}_2\right)^{q_2}}{\left(\lambda_1 \bar{K}_1^{\alpha_1}\right)^{q_1}}$ *into (13), we have* $\bar{T} = \lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2$, *which implies by Proposition 2 that* $\bar{C}_2 = 0$ *and* $\bar{C}_1 > 0$.

Claim 2 $s = \frac{\varphi}{1-\varphi} \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2}\right)^{q_2}}{\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1 - \Omega_2 \bar{K}_2\right)^{q_1}} \Rightarrow \bar{C}_1 = 0$ *and* $\bar{C}_2 = \lambda_2 K_2^{\alpha_2} + \lambda_1 K_1^{\alpha_1} - \Omega_1 K_1 - \Omega_2 K_2$. *The procedure is the same of Lemma 1. Substitute* $s = \frac{\varphi}{1-\varphi} \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2}\right)^{q_2}}{\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1 - \Omega_2 \bar{K}_2\right)^{q_1}}$ *into (13): we have* $\bar{T} = -\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)$, *which implies, by Proposition 2 that* $\bar{C}_2 = 0$ *and* $\bar{C}_1 > 0$.

Hence, by construction, $s \in (s_{LOW}, s_{UP}) \Rightarrow \bar{C}_1 > 0 \wedge \bar{C}_2 > 0$. Is the converse true? ■

Proof of Corollary 3. Notice that when $s \to 0$ then $\bar{T} = \lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2$, and that when $s \to \infty$ then $\bar{T} = -\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)$. By Corollary 1 T is monotone, and by continuity, there exist a unique value of s at which $\bar{T} = 0$. Substituting $\bar{T} = 0$ into (13) and solving for $\hat{s}$, we obtain $\hat{s} = \frac{\varphi}{1-\varphi} \frac{\left(\lambda_2 \bar{K}_2^{\alpha_2} - \Omega_2 \bar{K}_2\right)^{q_2}}{\left(\lambda_1 \bar{K}_1^{\alpha_1} - \Omega_1 \bar{K}_1\right)^{q_1}}$. ■